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# A Theoretic Analysis of Combined Arms Teaming

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## **ABSTRACT**

This study presents a foundation for the comparative analysis of the various combined arms teaming in a simulated environment. The study consists of three stages. First, a discrete-event combat-simulation model of two opposing generic combined arms teams is developed. This model is used to study the relationships between six key attributes of combined arms teams: communication; detection; lethality; mobility; protection; and sustainment. Second, a genetic algorithm is embedded within the combat-simulator to evolve strategies for combined arms teams against a static opposing force. Finally, a two-population genetic algorithm is used to coevolve two opposing forces against each other. Games theory is used to analyse the results and to provide advice on the impact of adding, removing and replacing assets or capabilities within the teams. We conclude that diversity and specialisation within combined arms teams is essential to the Land force. Furthermore, no single combined arms team is sufficient to ensure a tactical victory on the battlefield against all potential opponents. A range of different options for constructing combined arms teams is required.

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## Executive Summary

Land Operations Division is undertaking research into enduring battlespace effects, in particular the effects that the Hardened Networked Army is required to deliver and sustain, in support of the JP 5000 Networking the Land Battlespace project. This study presents an analysis of the interactions that occur within combined arms teams at the tactical level and the emergent battlefield effects that are produced as a result of these interactions.

An agent-based approach is used to implement a combat simulator for the study of combined arms teaming. Six key capabilities of combined arms teams and their members are modelled as attributes of agents in simulations. These attributes are: lethality; mobility; protection; detection; communication; and sustainment. The interactions between the attributes are studied at the force level and at the agent level. It was determined that lethality, protection and sustainment had the greatest impact on the outcome of combat at the force level, with detection and communication having lesser impact. The usefulness imparted by mobility could not be determined in this study. At agent level, detection was found to be the most important attribute. These results appear to conflict. However, there is no logical inconsistency because the two methods apply to different situations. Problems interpreting the results arise not from the model itself, the data or the techniques of analysis but from the inherent complexity of the problem. In particular, the synergistic nature of interactions within combined arms teams is difficult if not impossible to quantify. However, both approaches provide insight into the nature of combined arms teaming and are therein useful in their own right.

Games theory is used to provide a quantitative valuation of the consequences of adding, removing and replacing capabilities within combined arms teams. This approach measures the value of potential combined arms teams loosely based on the sensitivity of the model to changes within sets of candidate options. For example, given a number of options for combined arms teams, the criticality of each option can be determined by removing those options from one of the forces available strategies and measuring the change in the value or outcome of the game. This technique demonstrates the necessity of maintaining and fielding a number of different configurations of combined arms teams and has the potential to provide valuable insights into the effectiveness of combined arms teams of various compositions for the Land force.

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# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Battle Simulator</b>	<b>2</b>
2.1	Agents and Attributes . . . . .	2
2.2	Phases and Actions . . . . .	4
2.3	Simulation . . . . .	6
<b>3</b>	<b>Genetic Algorithm</b>	<b>6</b>
3.1	Agents, Teams and Populations . . . . .	6
3.2	Single-Population Genetic Algorithm . . . . .	7
3.3	Genetic Operators . . . . .	8
3.4	Two-Population Genetic Algorithm . . . . .	9
<b>4</b>	<b>Empirical Results</b>	<b>10</b>
4.1	Parameter Space Exploration . . . . .	10
4.2	Evolved Strategies . . . . .	15
4.3	Coevolved Strategies . . . . .	18
<b>5</b>	<b>Discussion</b>	<b>22</b>
<b>6</b>	<b>Conclusions</b>	<b>23</b>
<b>7</b>	<b>Acknowledgements</b>	<b>24</b>
	<b>Appendix A: Input Parameters</b>	<b>27</b>
	<b>Appendix B: Simulation Data</b>	<b>28</b>
	<b>Appendix C: Games Theory</b>	<b>33</b>
A.1	Two-Person Zero-Sum Games . . . . .	33
A.2	Dominated Strategies . . . . .	33
A.3	Saddle Points . . . . .	34
A.4	Mixed Strategies . . . . .	34



# 1 Introduction

The Australian Defence Force is undertaking a study of the current and future capabilities of the Land force, the potential benefits and limitations these capabilities bring to the Land force, and the effects produced by various combinations of these capabilities at the tactical and operational level (Commonwealth of Australia, 2004a). To guide the development of Land's capabilities as its underlying requirements, needs and imperatives evolve over a thirty year timeframe in an ever-changing geopolitical environment, the ADF "must be able to articulate concepts and effects to meet the range of potential conflicts, operational environments and Military Strategic Objectives that are currently uncertain." (Curtis and Dortmans, 2001, p.364). This paper studies the relationship between the capabilities of the Land force and the fundamental characteristics, attributes and skills inherent to parties within the force which enable, facilitate and support the production, application and sustainment of effects in the tactical Land environment.

Perhaps the first simulated study of Land's capabilities and the relationship of these capabilities to a conceptual model of the Army as a set of enabling core skills was conducted by Shine (2004a,b, 2005). In this study, the conceptual model of the Army as the set of seven army-as-a-system core skills; namely engagement, information collection, sustainment, communication, protection, movement and decision making, as developed by Curtis and Dortmans (2001); was used to create a set of behaviours and characteristics for the Land force. MANA (Lauren et al., 2001), an agent-based combat simulator developed by the New Zealand Defence Technology Agency, was then used to model and simulate these behaviours and characteristics by defining the Land force as a collection of interacting agents. In this manner, relationships between the behaviours of the agents in MANA and the army-as-a-system core skills were defined. For example, the behaviour "better fire-power", which increased agents' probabilities of killing enemy agents by a multiplier of 10, was related to the army-as-a-system core skill engagement. Combined arms teams with the better firepower tag could then be matched against, for example, combined arms teams with the "flanking attack" tag, which required information collection and movement. This approach was limited to defining specific instances of relationships between core skills and behaviours in agents rather than general relationships between core skills and agents' capabilities. Furthermore, because the core skills and the behaviours in MANA were related to each other by subjective interpretation rather than as characteristics inherent to agents, the appropriateness of the relationships is questionable.

A subsequent, independent study of combined arms teaming was conducted by Baker (2004), Baker *et al.* (2004) and Botting (2004). This study proposed a set of five core skills and defined these core skills as attributes of agents in combined arms teams. Agents' abilities in the core skills were rated on an ordinal scale between 1 and 15. For example, agents with a lethality rating of 1 to 15 represented agents with weapons' range of 1 to 15 units respectively. Each agent was provided with a total of 15 points to distribute over the five core skills. This implementation of core skills lead to an intractably huge space of possible combined arms teams. To search this space, Baker *et al.* applied an evolutionary search algorithm. More specifically, a genetic algorithm was employed.

Evolutionary algorithms are commonly applied as biased random search methods to provide heuristic solutions in optimisation problems; see Alander (1995), Bodnovich and Wong

(1998) and De Jong (1993). All paradigms in the class of evolutionary algorithms share a conceptual design rooted in the principle of natural evolution, see Darwin (1859) and Spears et al. (1993). Individual structures adapt according to a complex evolutionary process modelled by a sequence of selection, reproduction and mutation operations. A generation of such individual structures forms a basis for a simulation model in which population pressure and competition are an instrument for natural selection and evolution, see Forest (1993). The genetic algorithm, see Goldberg (1989) and Holland (1975), is a type of evolutionary algorithm.

This study extends and completes Baker *et al.*'s approach by providing a meaningful foundation for the analysis of the results of the genetic algorithm. A discrete-event combat-simulation model of two opposing genetic combined arms teams is developed in Section 2. This model is used to study the set of six core skills or attributes: communication; detection; lethality; mobility; protection; and sustainment. The genetic algorithm is described in Section 3. This algorithm is used to coevolved strategies for the composition and capabilities of combined arms teams for the two opposing forces. Results are presented in Section 4 and games theory, see Appendix C, is used to analyse a statistically significant number of Monte Carlo combat-simulations of each pair of strategies. Refer to von Neumann (1928), von Neumann and Morgenstern (1944) and Nash (1950, 1951) for the original development of games theory. A discussion of this studies findings and its' conclusions are presented in sections 5 and 6 respectively.

## 2 Battle Simulator

### 2.1 Agents and Attributes

Agents are characterised by a set of six attributes that represent the agents' capabilities in communication, detection, lethality, mobility, protection and sustainment. These attributes, rated on the ordinal scale with a minimum value of unity, affect how agents interact and the behaviours they exhibit as described below. These behaviours are designed to be as simple enough to model and implement yet sufficiently complex to give reasonable results for analysis.

- *Communication.* An agent's communication attribute defines the maximum number of distinct communications that can be conducted simultaneously. This value, when multiplied by 10, also defines the probability that each independent communication is successfully transmitted. For example, a value of 5 describes an agent capable of transmitting a total of 5 reports or packets of information. Each of these reports has a 50% probability of successfully being transmitted. The agent is aware of the success or failure of each attempted communication and will retransmit the same report should it have excess bandwidth. The information contained within a report and the ways other agents make use of this information are described in Section 2.2.
- *Detection.* An agent's detection attribute defines the maximum range over which enemy agents can be sensed. This value, when multiplied by 10, also defines the probability that enemy agents are successfully detected between one-thirds maxi-



mum and maximum range. At ranges between zero and one thirds of the maximum range, enemy agents are always successfully detected. For simplicity, no distinction is made between detecting, identifying and recognising. Agents are either detected or not. For example, a value of 5 describes an agent capable of detecting an enemy within a sensor circle of radius 5 centered on the sensing agent. The enemy agent is successfully detected with 100% probability at a distance less than  $5/3$  units and 50% probability at a distance between  $5/3$  units and 5 units. Agents' situational awareness and the actions taken upon the detection of enemy agents are described in Section 2.2.

- *Lethality.* An agent's lethality attribute defines the maximum range over which enemy agents can be prosecuted. This value, when multiplied by 10, also defines the probability that enemy agents are successfully engaged between two-thirds maximum and maximum range; that is, the probability that the enemy agent is damaged or destroyed. At ranges between zero and two-thirds of the maximum range, enemy agents are always successfully engaged. For example, a value of 5 describes an agent capable of prosecuting an enemy within an engagement circle of radius 5 centred on the engaging agent. The enemy agent is successfully damaged or destroyed with 100% probability at a distance less than  $10/3$  units and 50% probability at a distance between  $10/3$  units and 5 units. The classification of an enemy lying within the engagement circle as a valid or invalid target and the consequences of a successful prosecution are described in Section 2.2.
- *Mobility.* An agent's mobility attribute defines the maximum range over which the agent can manoeuvre when given the opportunity to move. For example, a value of 5 describes an agent capable of manoeuvring over a distance of 5 units across the battlefield. Terrain is not modelled in this study. How agents move and the movement algorithm itself are described in Section 2.2.
- *Protection.* An agent's protection attribute defines the number of times the agent can sustain damage before being destroyed. For example, a value of 5 describes an agent capable of surviving prosecution a total of 5 times. For simplicity, agents' capabilities are not diminished in any way by damage. Agents are either destroyed or functional. The protection attribute can be interpreted as the amount of physical armour carried by the agent, as a numerical size representing an agent comprised of more than a single individual, or a combination of the two.
- *Sustainment.* An agent's sustainment defines the total number of times that the agent may scan for enemies and operate its radio or employ its weapon; that is, attempt an action leading to the use of both the detections and communications attributes in sequence once or the lethality attribute once. For example, a value of 5 describes an agent capable of scanning for enemies and operating its radio in sequence or employing its weapon up to a total number of 5 times when given the opportunity to act. The agent randomly chooses either to both scan for enemy and communicate or use its weapon. Agents will not employ their weapons if no enemy targets are detected. The implications of sustainment and the benefits it provides to an agent in relation to communication and lethality are described in Section 2.2.

## 2.2 Phases and Actions

Agents act in accordance with a sequence of set phases of action. Listed in order, these phases of action are: a manoeuvre phase, a detection phase, an engagement phase, a communications phase and a sustainment phase.

- *Manoeuvre Phase.* Agents' positions are real valued and as such the locations of agents are not limited to discrete locations on a grid. Movement is conducted by first identifying a destination point to move towards. This point is chosen randomly, with equal probability, from three options as follows.
  1. Move towards your own (friendly) force's center of gravity.
  2. Move towards the opponent (enemy) force's center of gravity.
  3. Move towards the mission objective.

The friendly and enemy centers of gravity are defined in a similar manner to that used by MANA - that is, a simple average of the  $x$  and  $y$  values of all friendly and enemy agents positions respectively. The center of gravity for a force without any constituent members is undefined. The mission objective, for all agents irrespective of which force they belong to, is set to the fixed point  $(x_{OBJ}, y_{OBJ})$  depending on the scenario. After selecting a point to move to, a straight-line segment is extended between this point and the current location of the agent. The agent then moves along the line up to a maximum distance defined by its mobility attribute. These rules are specifically designed to be the simplest set of coherent behaviours appropriate for the simulation set-up described in the Section 2.3, in which the blue and red forces take up position in symmetrically opposing sides of the battlefield with the objective to capture and hold the middle of the board. This does not prohibit other rules being implemented or studied at a later date, for example including friendly and enemy force numbers and distance to mission objective for example. The three manoeuvre phase behaviours broadly encourage the two forces to work collectively as a closely coupled whole. Such behaviours would not be suitable in modelling paratroopers, for example, who are dropped behind enemy lines. In such a scenario, specific rules to govern the behaviour of detached forces are necessary to prevent the paratroopers from an effectively suicidal attraction to their own forces across the other side of the battlefield. This behaviour could be formed by modifying rule 1, for example, to an weighted attraction favouring the closest friendly agents. A further sensible change, to rule 2 now, is for the paratroopers to be attracted to a nearby enemy agent who is furthest from the enemy center of gravity rather than the enemy center of gravity itself. Hence, the small and detached paratrooper force could focus on destroying vulnerable and unsupported enemy agents rather than fighting the enemy's main effort without the support of other friendly agents.

- *Detection Phase.* Agents maintain local situational awareness by sweeping the surrounding area for enemy agents during the detection phase. A single pass is conducted to locate enemies as explained earlier when describing the detection attribute. Furthermore, a global list of enemy agents that have been detected and subsequently reported by other friendly agents is available to supplement the agent's situational awareness. The global list of enemy agents is further explained in the communica-

tions phase below. Agents are aware of whether or not their comrades are aware of the same enemy agents that they themselves have knowledge of. Enemy agents are marked as reported or unreported accordingly.

- *Engagement Phase.* Agents prosecute enemies according to the rules explained earlier when describing the lethality attribute. Valid targets for prosecution are those whose locations are known - that is, targets within the agents' local or global situational awareness lists. All other enemy agents are invalid targets and cannot be engaged. Since targets in the global situational awareness list are valid then, assuming the targets are also within the range identified by the lethality attribute, targets outside sensor range can be prosecuted. Hence, agents may act as spotters or forward observers for their force.
- *Communications Phase.* Upload of information is assumed to be either a manual or automatic process over a carrier susceptible to failure, for example, sending a Variable Messaging Format (VMF) transmission over a combat net radio or Battle Management System (BMS) by line-of-sight or range extension via relay. Download of information is assumed to be an automatic process over a carrier that is not susceptible to failure in open terrain, for example, receiving blue-force-tracker information and known red-force locations facilitated by a BMS over a secure satellite. Agents communicate according to the rules explained earlier when describing the communications attribute. Only one type of report is modelled in this study, namely an enemy-sighting report. This report, if successfully transmitted by an agent, is distributed to all friendly agents in the form of the global situational awareness list. Agents only report the location of enemy agents if they are not a part of the global situational awareness list. For simplicity, the location of the enemy agents in the global situational awareness list is assumed to be 100% reliable and exact. Knowledge of an enemy's location is lost as soon as that enemy enters a movement phase and the agent is removed from the opposing force's global situational awareness list.
- *Sustainment Phase.* As explained earlier when describing the sustainment attribute, an agent may repeat both the detection and communications phases or repeat the engagement phase depending on the value of their sustainment attribute. Sustainment is best interpreted as enabling agents to operate equipment at maximum efficiency. As such, it represents a rate of use which is likely to denote a combination of training and experience, characteristics of the hardware and software systems as well as the level of combat service support and resupply. A high level of sustainment does not necessarily offset say a low value of engagement or communications. For example, a lethality value of 1 means that the agent can only prosecute enemy at close range and has only a 10% probability of damaging opponents. Hence, even if the agent sustained fire 10 times there is still a  $100 * 0.910 \approx 35\%$  probability that the enemy is not damaged. Hence, a lethality value of 10 is far more desirable. However, sustaining fire say 3 times with a lethality level of 7 is likely to be preferable as it provides almost certain success when focussed on a single enemy agent and is very likely to actually damage the enemy more than once. However, it is also important to note that the area over which enemies can be engaged drops dramatically as the lethality value decreases. For example, the engagement area at a lethality value of

10 is  $\pi * 102 \approx 314$  units squared but the engagement area at a lethality value of 7 is less than half that at  $\pi * 72 \approx 153$  units squared.

## 2.3 Simulation

Battles are conducted on a two-dimensional 100 by 100 units battlefield as follows. First, a blue and a red force of  $N$  agents each are initialised with random attributes adding to  $P$  such that each attribute has value between 1 and 10 inclusive and each attribute is a whole number. Members of the blue force are placed upon the battlefield randomly with  $(x, y)$  coordinate value such that  $x \in [0, 100]$  and  $y \in [80, 100]$ . Members of the red force are placed upon the battlefield randomly with  $(x, y)$  coordinate value such that  $x \in [0, 100]$  and  $y \in [0, 20]$ . Hence, the blue force takes up position in the battlefield in the bottom 20% of the board and the red force in the top 20% of the board. The objective point, for both the blue and red forces, is set to the centre of the battlefield  $(x_{OBJ}, y_{OBJ}) = (50, 50)$ . The centre of the battlefield is chosen for convenience. Selecting an objective within or close to one force's original placement region may affect the outcome of combat because that force is more compact than the opposing force who must advance to the objective, which causes increased separation in the force.

Agents, both blue and red, behave according to the rules explained in Section 2.2. The simulation is conducted as follows. The first blue agent is allowed an action phase followed by the first red agent, then the second blue agent and second red agent and so on until the last blue agent and last red agent, upon which the process starts anew. Agents that are destroyed are excluded from any influence on either the blue or red forces and are prohibited from acting. The initial placement of the blue and red forces effectively rules out the possibility of interacting with the enemy in the short term. Hence, there is no benefit obtained or bias introduced from blue acting first rather than red. The fact that the blue and red agents alternate between action phases means that the bias introduced by other discrete-time combat-simulators, in which the entire blue force acts before the red force can respond, is eliminated. Simulations are terminated upon the events that each undestroyed agent has acted  $A$  times or the event that the blue or red force has been completely destroyed.

# 3 Genetic Algorithm

## 3.1 Agents, Teams and Populations

In this study, the foundation of combined arms teams are single units or agents. Details of what exactly agents are composed of, their attributes and representations are provided in Section 2. Further to this description, denote the set of all possible agents by  $\mathcal{A}$ . With an agent as a basic building block, we define the concept of a combined arms team as an ordered, finite and non-empty collection of agents. We identify individual agents in a team by the ordinal enumeration  $\{1, 2, \dots N\}$  over the Natural Numbers  $\mathbb{N}$ , where  $N$  is the number of agents in a team.

Denote the set of all possible distinct combined arms teams by  $\mathcal{T} \subset \mathcal{A}^N$ . We define an ordered, finite and non-empty collection of teams as a population. The distinct teams in a population are identified by the ordinal enumeration  $\{1, 2, \dots, T\}$  over the Natural Numbers  $\mathbb{N}$ , where  $T$  represents the number of teams in a population. The population then denotes a collection of candidate solutions for combined arms teams.

We associate with each team  $t \in \mathcal{T}$  a random variable  ${}_tX_{OPFOR}$ , conditional on the opposing-force team  $OPFOR$  which is itself a member  $\mathcal{T}$ . This random variable returns a real number which denotes the fitness or payoff that the combined arms team  $t$  receives on the outcome of  $B$  independent, stochastic and simulated battles against the opposing-force team  $OPFOR$ . The simulation of a battle is discussed in Section 2.

Let there be  $P$  points to distribute between  $K$  attributes, where  $P \geq K$ . Then the total number of distinct agents, in the set of possible agents  $\mathcal{A}$  having no attribute less than unity, is implicitly given as the solution to

$$x_K + x_{K-1} + \dots + x_2 + x_1 = P - K, \quad (1)$$

where  $x_i \in \mathbb{N}$ ,  $i = 1 \dots K$ .

Counting the number  $\nu$  of solutions to equation (1) is equivalent to counting the number of possible ways to allocate  $P$  identical objects to  $K$  distinct bins. This problem has solution

$$\nu = \binom{P-1}{P-K}. \quad (2)$$

The number  $v$  of distinct combined arms teams composed of  $N$  agents, or the number of ordered collections of  $N$  agents in which agents may appear more than once, is then

$$v = \sum_{N=1}^{\nu} \sum_{m=N}^{\nu} \dots \sum_{b=c}^{\nu} \sum_{a=b}^{\nu} 1. \quad (3)$$

The total number of distinct combined arms teams - that is, the cardinality of  $\mathcal{T}$ , is so intractably huge for the parameter set given in Appendix A that it could not be computed using equation (3) on a Pentium IV processor. Hence, to search this space for desirable combined arms teams, matching some set number of criteria yet to be defined, a heuristic algorithm is necessary. The heuristic algorithm we use, following the work of Baker (2004), is the genetic algorithm.

## 3.2 Single-Population Genetic Algorithm

The genetic algorithm is a discrete-time process which conducts a biased random search over a solution space by applying a sequence of genetic operators. We base our genetic algorithm loosely on Goldberg (1989) and Holland's (1975) seminal work on genetic algorithms and operators and Baker et al.'s (2004) study of combined arms teaming but observe that our particular adaptation is a new and unique interpretation of their original formulations.

The genetic algorithm maintains at each time step  $g = 0, \dots, G$  a population of combined arms teams, where  $G$  represents the number of generations produced by the genetic algorithm.  $G$  is then an iteration counter. The index  $G$  uniquely identifies each population by generation, where the term generation is interpreted in the context of the genetic algorithm as the time of birth of a society and not merely a mechanical process producing an output. A sequence of populations is generated according to a number of stochastic rules implemented in the form of genetic operators.

The genetic algorithm is described as follows, with an exact description of each operator following in the next section. At time  $g = 0$ , the population  $p_0$  of combined arms teams is generated randomly with uniform distribution from  $\mathcal{T}$ . An opposing-force team *OPFOR* is taken as input. This team represents the force against which the genetic algorithm battles in order to determine the fittest combined arms teams and hopefully, under the process of natural selection, evolve the population such that only the strongest and fittest teams survive. At each subsequent time step  $g \geq 1$ , the fitness of all teams in the population  $p_{g-1}$  is calculated. This calculation is performed, as previously discussed, through empirical simulation of  $b = 1, \dots, B$  independent battles matching each team  $t$  in the population against the opposing-force team *OPFOR*. The numerical fitness value assigned is

$$\frac{1}{B} \sum_b [c_{OPFOR}(b) - c_t(b)], \quad (4)$$

where  $c_t(b)$  and  $c_{OPFOR}(b)$  denote the number of casualties sustained by teams  $t$  and *OPFOR* respectively in the  $b^{\text{th}}$  battle. The time to secure the objective point is not a factor in this fitness calculation. A selection operation then operates on the old population  $p_{g-1}$  to construct a pool of teams  $q_g$  called the mating pool. The genetic operators crossover, mutation and elitism are applied to the mating pool  $q_g$ . The resultant pool becomes the new population  $p_g$ . This process is repeated  $G$  times upon which time the process is terminated. At the end of this process, a team with highest fitness from the population  $p_G$  is taken to be the combined arms team best suited to defeat the *OPFOR* team.

### 3.3 Genetic Operators

We split the genetic operators into three stages, based on the type of operator and their interactions with the mating pool  $q_g$  and the populations  $p_{g-1}$  and  $p_g$ . The first stage operator selection acts on the population  $p_{g-1}$  to generate the mating pool  $q_g$ .

1. *Selection.* Two teams are chosen at random from the population  $p_{g-1}$ . The team with highest fitness, breaking ties in fitness randomly, is inserted into the mating pool  $q_g$ . A mating pool of size  $N$  is constructed by repeating this process  $N$  independent times.

Two operators, crossover and mutation, operate on the mating pool  $q_g$  during the second stage of time step  $g$ .

2. *Crossover.* Two teams  $t_1$  and  $t_2$  are chosen at random from the mating pool  $q_g$ . An agent  $a_1$  is chosen at random from  $t_1$  and the corresponding agent  $a_2$  is chosen from  $t_2$  - that is, the agent  $a_2$  with the same ordinal index as  $a_1$ . With probability

$p_{CROSS}$ , the agents  $a_1$  and  $a_2$  are swapped so that agent  $a_1$  takes the place of agent  $a_2$  in  $t_2$  and agent  $a_2$  takes the place of agent  $a_1$  in  $t_1$ . Hence, with probability  $1 - p_{CROSS}$  no action is taken. This step is repeated a total of  $\lfloor NT/5 \rfloor$  times.

3. *Mutation.* Every agent  $i$  in every team the mating pool  $q_g$  is enumerated over. With probability  $p_{MUTATE}$ , agent  $i$  is replaced by a new agent generated randomly with uniform distribution from  $\mathcal{T}$ .

In the final stage of time step  $g$ , the operation elitism operates on the mating pool  $q_g$  and population  $p_{g-1}$  to construct the population  $p_g$ .

4. *Elitism.* The fittest team  $t_f$  in the population  $p_{g-1}$ , splitting ties between fittest individuals, and a team  $t_r$  chosen at random in the mating pool  $q_g$  are chosen. The team  $t_r$  in  $q_g$  is replaced by the team  $t_f$  so that the fittest team from the previous time step  $g - 1$  always exists in the mating pool  $q_g$ . The entire mating pool  $q_g$  is then copied into  $p_g$  so that the mating pool  $q_g$  becomes the new population  $p_g$ .

### 3.4 Two-Population Genetic Algorithm

Up to this point, we have discussed the application of a genetic algorithm which evolves a population, for the blue force say, against a static opposing-force team, the red force say. Such an implementation is useful in determining a blue force combined arms team best suited to defeat a given red force team. However, one may equally well ask what happens when the red force is also simultaneously evolved against the blue force. A coevolving implementation of the genetic algorithm involving two populations is required.

We begin explaining our implementation of the two-population genetic algorithm by stating that the fundamental operations involved in this genetic algorithm are no different to those applied in the single-population genetic algorithm, nor is the sequence of the algorithm itself any different. Let us start by introducing the concept of running two single-population genetic algorithms  $GA_1$  and  $GA_2$ , for the blue force and red force respectively, completely independently of each other apart from the fact that they each run simultaneously such that at any instance in time  $t \geq 0$ , both  $GA_1$  and  $GA_2$  are in time step  $g_1$  and  $g_2$  respectively with  $g_1 = g_2 = t$ . Now at time  $t$ , we ask the genetic algorithms  $GA_1$  and  $GA_2$  to elect a team  $e_{1,t}$  and  $e_{2,t}$  respectively from the populations  $p_{1,t}$  and  $p_{2,t}$  respectively denoting the populations evolving within  $GA_1$  for the blue force and  $GA_2$  for the red force, to represent the blue force and red force respectively at that instance in time. We assert that the blue and red forces elect their teams rationally such that a team with highest fitness is chosen. The teams elected at  $t = 0$ , before any battles have been played and hence before any fitness values are defined, are chosen randomly. We now remove our assumption of the two single-population genetic algorithms run independently of each other and formally define the two-population genetic algorithm as merely an interlacing of these two single-population genetic algorithms running concurrently with a single important modification - namely that, the single-population algorithms do not evolve against a static opposing-force. Instead, at each time step  $t \geq 1$ ,  $GA_1$  evolves against its opposing-force's elected team  $e_{2,t-1}$  and  $GA_2$  evolves against its opposing force's elected team  $e_{1,t-2}$ . Hence, at each time step, the two forces continually coevolve against the opposing-force pitted against them in the last time step. That is, they don't know *a priori* the exact

composition of the team they will battle in the current time step  $t$  but do know the composition of the team that was elected to face them in the previous time step  $t - 1$  and use this knowledge to evolve their force.

## 4 Empirical Results

### 4.1 Parameter Space Exploration

In this section, a limited study of the interactions between the six attributes is conducted.  $R = 500$  simulations are conducted between two distinct forces in which each force is enhanced, from an initial balanced force in which each agent has  $P/6 = 3$  points for each attribute, by adding a single point to 2 of the attributes of every agent in the force. There are 21 ways to distribute 2 points amongst 6 attributes, including variations in which the two points are added to the same attribute.  $R$  simulations are conducted for each of the 210 unique pairs of the 21 possible forces to calculate the fitness of the each simulation. Recall that, to calculate the fitness of a simulation,  $B = 20$  battles are conducted. Hence,  $BR = 10,000$  independent battles are conducted for each of the 210 unique pairs. The mean fitness and variation over the  $R$  simulations is documented in Appendix B together with a 99.9% two-sided confidence interval for the sample mean.

Simulation results are tabulated in Figure 1. In this figure, the mean fitness is entered into the matrix to 1 significant digit. Rounding the results to 1 digit is entirely safe when it is observed that the greatest variance over all results is approximately 0.50 so that the standard deviation of all results is strictly less than 0.71. The  $t$ -statistic for a two-sided 99.9% confidence interval about the sample mean at  $R = 500$  degrees of freedom is approximately 3.3101. From Appendix B, the largest interval for the sample means is  $\pm 0.104706$ . Hence, we are certain within reasonable doubt that all results are statistically valid. When rounding results to 0, the signs ‘+’ and ‘-’ are preserved for interest. For example, the fitness of lethality against protection + communication from Appendix B is -0.445750. This value is recorded as -0 in Figure 1, denoting that a force enhanced by augmenting agent’s lethality attributes twice defeats a force enhanced by augmenting agent’s protection and communication attributes once each and the defeat occurs with a fitness value strictly more than -0.5 and strictly less than 0. The matrix is colour coded as follows. Scores of 2 or greater are coloured dark blue, scores of 1 are coloured light blue, scores of -1 are coloured light orange (yellow) and scores of -2 or less are coloured dark orange (yellow). The leading diagonal is shaded light grey.

A number of insightful comments can be made about Figure 1. Perhaps the most obvious of these is the poor performance of forces with enhancements in mobility. This is a direct consequence of the scenario and not necessarily a comment on the usefulness of enhanced mobility in general. The scenario under study specifically and intentionally models a battle in which two opposing forces encounter each respective opposing force without prior warning. The scenario is developed such that each side has sufficient time to form as a combined arms team, hence reducing the potential for ambush or strike in enemy territory. Enhanced mobility in this study merely means that the force attains the goal point at the center of the battlefield before its opponent. As a consequence, the point



	Lethality	Mobility	Protection	Detection	Communication	Sustainment	Lethality + Mobility	Lethality + Protection	Lethality + Detection	Lethality + Communication	Lethality + Sustainment	Mobility + Protection	Mobility + Detection	Mobility + Communication	Mobility + Sustainment	Protection + Detection	Protection + Communication	Protection + Sustainment	Detection + Communication	Detection + Sustainment	Communication + Sustainment
Lethality		2	-0	2	1	-0	1	-0	0	-1	-1	1	2	2	1	0	-0	-0	1	0	0
Mobility	-2		-3	-2	-2	-2	-1	-3	-3	-2	-3	-2	-1	-1	-1	-3	-3	-3	-2	-3	-2
Protection	0	3		2	1	-0	2	0	1	1	-0	2	3	2	1	0	-1	-0	2	1	0
Detection	-2	2	-2		-1	-2	0	-2	-1	-1	-2	-0	0	0	0	-1	-2	-2	-0	-1	-1
Communication	-1	2	-1	1		-1	0	-1	-0	-1	-1	0	1	1	1	-0	-1	-1	0	-0	-0
Sustainment	0	2	0	2	1		1	-0	0	0	-0	1	2	2	1	1	0	-0	1	0	0
Lethality + Mobility	-1	1	-2	-0	-0	-1		-1	-1	-1	-1	-0	1	1	0	-1	-1	-1	-0	-1	-1
Lethality + Protection	0	3	-0	2	1	0	1		1	1	-0	2	2	2	2	1	1	0	2	1	1
Lethality + Detection	-0	3	-1	1	0	-0	1	-1		-0	-0	1	2	1	1	0	-0	-1	1	0	0
Lethality + Communication	1	2	-1	1	1	-0	1	-1	0		-0	1	2	2	1	0	0	-0	1	0	0
Lethality + Sustainment	1	3	0	2	1	0	1	0	0	0		2	2	2	2	1	1	-0	1	1	1
Mobility + Protection	-1	2	-2	0	-0	-1	0	-2	-1	-1	-2		1	1	-0	-1	-1	-2	-0	-1	-1
Mobility + Detection	-2	1	-3	-0	-1	-2	-1	-2	-2	-2	-2	-1		-0	-1	-2	-2	-2	-1	-1	-1
Mobility + Communication	-2	1	-2	-0	-1	-2	-1	-2	-1	-2	-2	-1	0		-0	-1	-2	-2	-1	-1	-1
Mobility + Sustainment	-1	1	-1	-0	-1	-1	-0	-2	-1	-1	-2	0	1	0		-1	-1	-1	-1	-1	-1
Protection + Detection	-0	3	-0	1	0	-1	1	-1	-0	-0	-1	1	2	1	1		-0	-1	1	-0	-0
Protection + Communication	0	3	0	2	1	-0	1	-1	0	-0	-1	1	2	2	1	0		-1	1	0	-0
Protection + Sustainment	0	3	0	2	1	0	1	-0	1	0	0	2	2	2	1	1	1		2	1	0
Detection + Communication	-1	2	-2	0	-0	-1	0	-2	-1	-1	-1	0	1	1	1	-1	-1	-2		-1	-1
Detection + Sustainment	-0	3	-1	1	0	-0	1	-1	-0	-0	-1	1	1	1	1	0	0	-1	1		-0
Communication + Sustainment	-0	2	-0	1	0	-0	1	-1	-0	-0	-1	1	1	1	1	0	-0	-0	1	0	

Figure 1: Simulation results

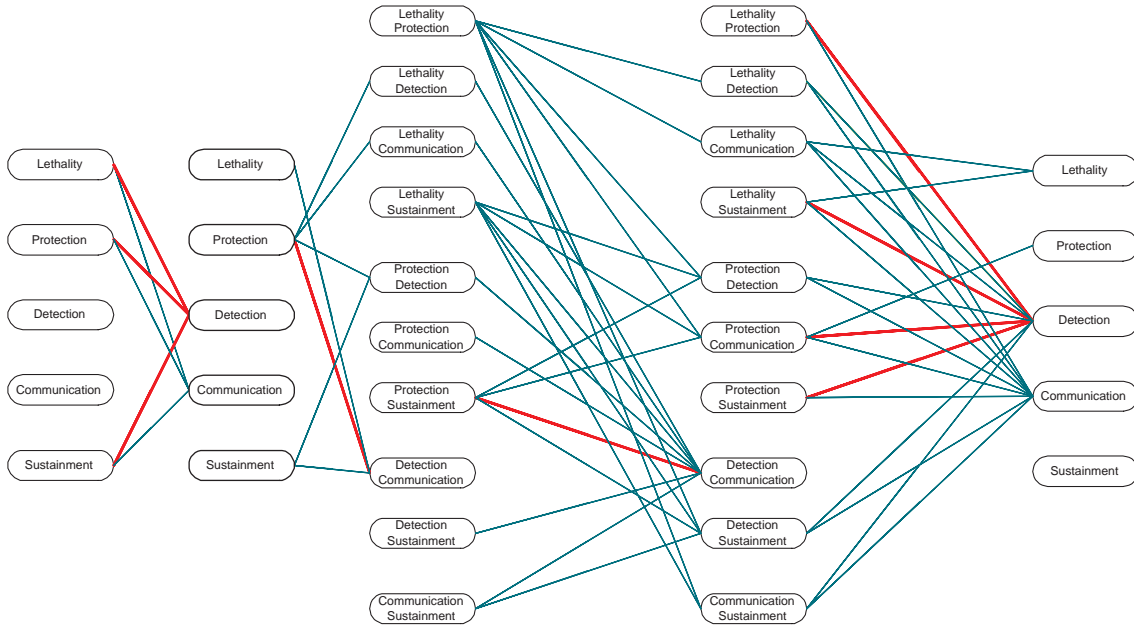


Figure 2: Defeats network

at which combat occurs is moved slightly from the center of the battlefield into enemy territory. For all practical intents and purposes, this scenario is insufficient to determine the relative benefits of enhanced mobility. However, such a study could be conducted by assigning a tactical benefit and liability, perhaps in combat and detection range, at different regions of the battlefield. For example, a hill could be marked as a tactical advantage and a valley as a tactical disadvantage. It is then important for each force to defend regions with a high tactical advantage and attack regions with a low tactical advantage. Mobility is critical for such manoeuvres.

Observe in Figure 1 the unusual way that attributes interact. For example, a force with enhanced lethality defeats a force with enhanced communications and a force with enhanced detection. However, a force with enhanced lethality is defeated by a force with enhancements to both lethality and communication and also by a force with enhancements to both lethality and detection. These results are explained by observing the complex and non-linear ways in which attributes combine. Figure 2 depicts the defeat of various combinations of forces. The figure is interpreted left to right such that two nodes are linked if the rightmost node is defeated by the leftmost. For example, the link between the lethality node and the detection node is interpreted as a force with enhanced lethality defeats a force with enhanced detection. Thick red links depict substantial defeats with a fitness of 2 (-2) or more (less) from Figure 1. Thin cyan links depict minor defeats with a fitness of 1 (-1) or more (less) from Figure 1. Battles with fitness of 0 or -0 from Figure 1 are not displayed. For clarity, all combinations of forces with enhancements to mobility are removed.

The results from Appendix B are analysed using the Analytic Hierarchy Process (AHP) (Saaty, 1977, 1980), a method of multiple pairwise comparisons for a finite number  $n$  of options. In the AHP, the  $n(n-1)/2$  preferences  $m_{i,j}$  for each unique pairwise comparisons between the two options  $i, j \in 1, \dots, n, j > i$ , are recorded on the real or integer numbers

over the range  $[-9, 1] \cup [1, 9]$ . Negative preference scores denote an aversion to the first option  $i$  in preference to the second option  $j$  and positive values denote a preference for the first option  $i$  in aversion to the second option  $j$ . The preferences are entered into a  $n \times n$  upper triangular matrix  $M$ . The leading diagonal of  $M$  is filled with unity and the lower triangular partition of the matrix filled to enforce the condition  $m_{i,j} \equiv -m_{j,i}$ . Finally, negative values are inverted and multiplied by -1 giving

$$M = \begin{bmatrix} 1 & m_{1,2} & m_{1,3} & \dots & m_{1,n-2} & m_{1,n-1} & m_{1,n} \\ 1/m_{1,2} & 1 & m_{2,3} & \dots & m_{2,n-2} & m_{2,n-1} & m_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1/m_{1,n-1} & 1/m_{2,n-1} & 1/m_{3,n-1} & \dots & 1/m_{n-2,n-1} & 1 & m_{n-1,n} \\ 1/m_{1,n} & 1/m_{2,n} & 1/m_{3,n} & \dots & 1/m_{n-2,n} & 1/m_{n-1,n} & 1 \end{bmatrix}. \quad (5)$$

In this study, the fitness scores  $f(i, j) = -f(j, i)$ , describing the fitness of the force  $i$  against the force  $j$ , are translated for the AHP as

$$m_{i,j} = \begin{cases} f(i, j) + 1, & f(i, j) \geq 0, \\ (1 - f(i, j))^{-1}, & f(i, j) < 0. \end{cases} \quad (6)$$

The AHP derives the relative priority  $w_i$ ,  $i = 1, \dots, n$  of the  $n$  options by solving

$$\sum_{j=1}^n m_{i,j} w_j = \lambda w_i, \quad (7)$$

$$\sum_{i=1}^n w_i = 1. \quad (8)$$

In this formulation, the entries  $m_{i,j}$  approximate the relative priority ratios  $w_i/w_j$ . The normalised principle right eigenvector  $w$  of  $M$  approximates the priorities of the options, see Figure 3.

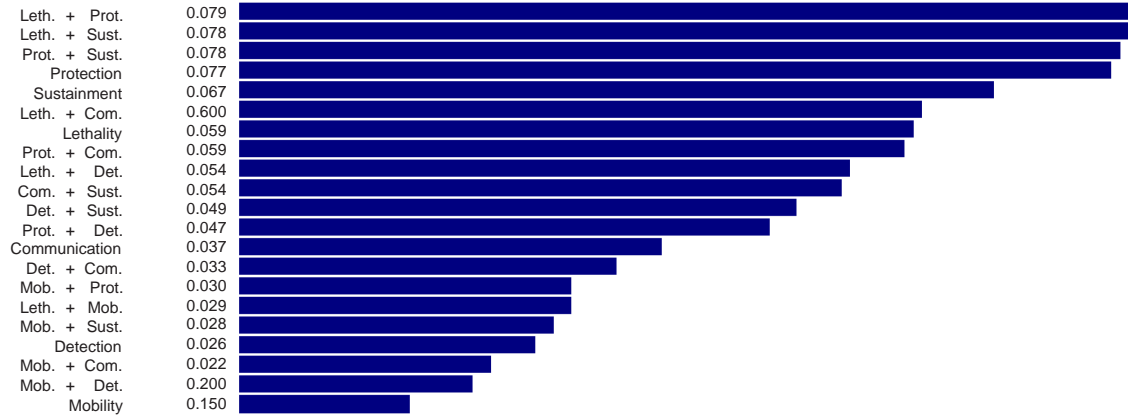


Figure 3: Comparative importance

In Figures 1, 2 and 3 the attributes broadly fall into three levels of effectiveness. Lethality, protection and sustainment are most important to a force. Next, communication and detection is important. Mobility is the least effective attribute in this scenario. However,

the ways in which attributes combine is interesting. For example, lethality alone is only ranked seventh in importance but the combinations of lethality with protection and sustainment are ranked first and second respectively. The reasons for these results are as follows.

- *Lethality.* Ultimately, a force must be able to inflict damage on the enemy to defeat it. Lethality directly promotes a force's ability to prosecute the enemy and is useful even when the force has poor detection and communication. Any detection and communication that does take place is acted upon with reasonable confidence of inflicting damage upon the enemy.
- *Protection.* The ability to survive contact with the enemy is paramount in all combat. High scores in all other attributes fails to negate the simple necessity to remain operational under fire.
- *Sustainment.* Sustainment increases an agent's ability to detect, communicate and prosecute. However, it is not a panacea and acts only to supplement sufficiently adequate natural attributes in agents and to minimise deficiencies in one or two of the agent's attributes. The outcomes of simulations are sensitive to the relationship between sustainment and agents' other attributes. For example, base scores of 2 substantially reduces the efficiency of sustainment while base scores of 4 substantially increases the efficiency of sustainment.
- *Communication.* Informing one's allies of the location of enemy agents is important to defeat those agents as quickly as possible and with minimal damage to friendly agents. Communications is a second order influence on the outcome of combat and is of less importance to the force if agents possess sufficient firepower and protection to defeat the enemy without assistance.
- *Detection.* A force must be able to detect enemy agents in order to prosecute them. However, being able to detect the enemy does not mean the agent has the firepower to engage and destroy that enemy, the communications to alert the force to the enemy's presence or the protection to survive the encounter. For these reasons, detection is of secondary importance.
- *Mobility.* Mobility does little to assist a force in this scenario.

The AHP defines a consistency index to measure the accuracy of  $w$  against random data. For this study the value

$$\frac{\lambda_p - n}{n - 1} = 0.0121735, \quad (9)$$

is obtained, where  $\lambda_p$  denotes the principle right eigenvalue of the  $n \times n$  matrix  $M$ . This value is divided by the average random consistency index, calculated from a sample of 500 matrices with randomly generated entries from the set  $\{1/9, \dots, 1/2\} \cap \{1, 2, \dots, 9\}$  satisfying  $m_{i,i} = 1$  and  $m_{i,j} = 1/m_{j,i}$ , to give a consistency ratio of approximately 0.008 or 0.8%. The consistency ratio for our results is substantially lower than the recommended 0.10 or 10% inconsistency limit and describes an overwhelming level of consistency in the results.

Mathematically, the matrix  $M$  is consistent if and only if  $m_{i,j}m_{j,k} = m_{i,k}$ , in which case it has principle eigenvalue  $n$  and equation (7) exactly solves  $Mw = nw$ . The concept of

consistency can be explained, in part, using the game Jan-Ken-Po. In this Jan-Ken-Po, two opponents simultaneously choose from one of three game strategies: rock; paper; and scissors. Scissors defeats paper and paper defeats rock. However, rock also defeats scissors, which is an anomaly in the AHP because if one is presented with three options  $A$ ,  $B$  and  $C$  and it is known that  $A$  is preferred over  $B$  and that  $B$  is preferred over  $C$  then it is logical to deduce that  $A$  is also preferred over  $C$ . This is called transitivity. The concept for consistency also includes the magnitude or degree for which options are preferred over others.

The consistency ratio of  $M$  is at odds with Shine's (2004a) study in which a broad skill set defeats engagement and decision making, engagement and decision making defeats information collection and movement, and information collection and movement in turn defeat a broad skill set. Our consistency result suggests that there exists a single best force, composed of identical agents, which defeats all other possible forces which are likewise composed of identical agents. However, it is easy to observe from Figure 1, that no such force exists within the proposed 21 forces. Lethality coupled with protection is the preferred force option but a zero-sum game  $\mathcal{G}$  with payoff matrix as presented in Figure 1, and with additional payoff values of 0 on the leading diagonal, has no saddle point solution. No inference can be made about forces other than the 21 force options examined in this section.

## 4.2 Evolved Strategies

In Section 4.1, we present an overly simplistic model designed only to study simple homogeneous forces, which do not represent combined arms teams. To examine such teams, we relax the constraint on the blue force and allow it to evolve against various red forces according to the genetic algorithm presented in Section 3 with input parameters as in Appendix A. This allows us to examine true combined arms teams or heterogeneous forces, in which agents assume diverse roles and functions within their teams.

Two cases are considered.

1. The blue force evolves against a balanced red force consisting entirely of agents having attribute scores of  $P/6 = 3$ .
2. The blue force evolves against a randomly generated red force initialised at the start of the simulation.

$R = 500$  independent replications of each of the two cases above are conducted. The mean fitness of the blue combined arms teams across the population of  $T = 10$  combined arms teams in the genetic algorithm and the fitness of the best individual combined arms team in the population is recorded for each replication. The mean and standard deviation across the  $R$  replications is displayed in Figure 4 and Figure 5.

Interestingly, the average mean-fitness for the blue force at generation  $g = 0$  is approximately -0.335 for case (1). This indicates that a randomly generated force, as is the blue force at  $g = 0$ , is defeated by a balanced force.

Let  $A_y(x)$ ,  $y \in \{L, M, P, D, C, S\}$ , denote the respective lethality (L), mobility (M), pro-

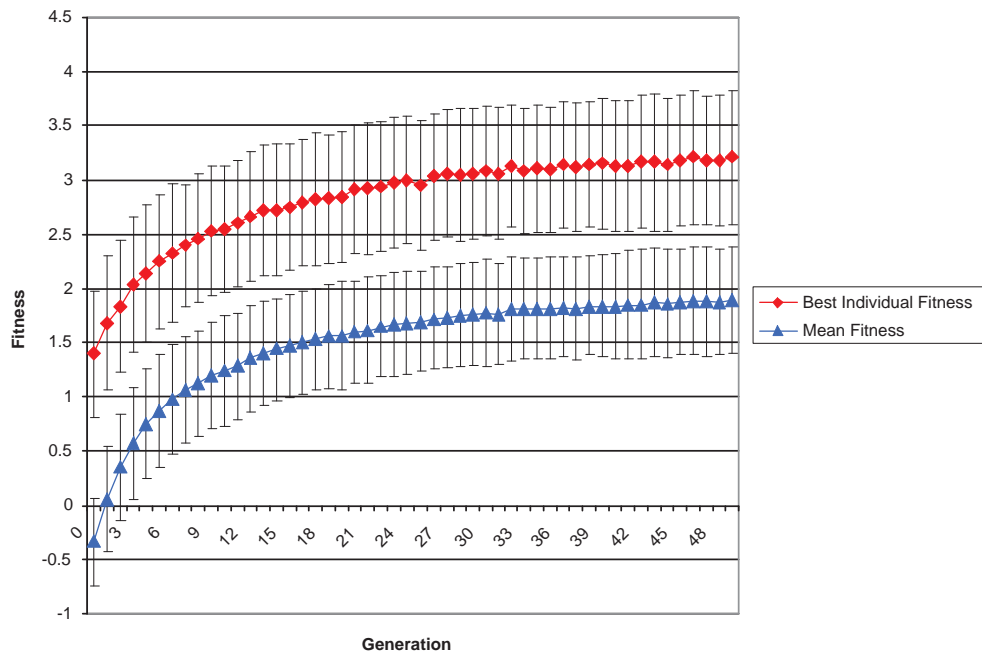


Figure 4: Evolution against balanced force

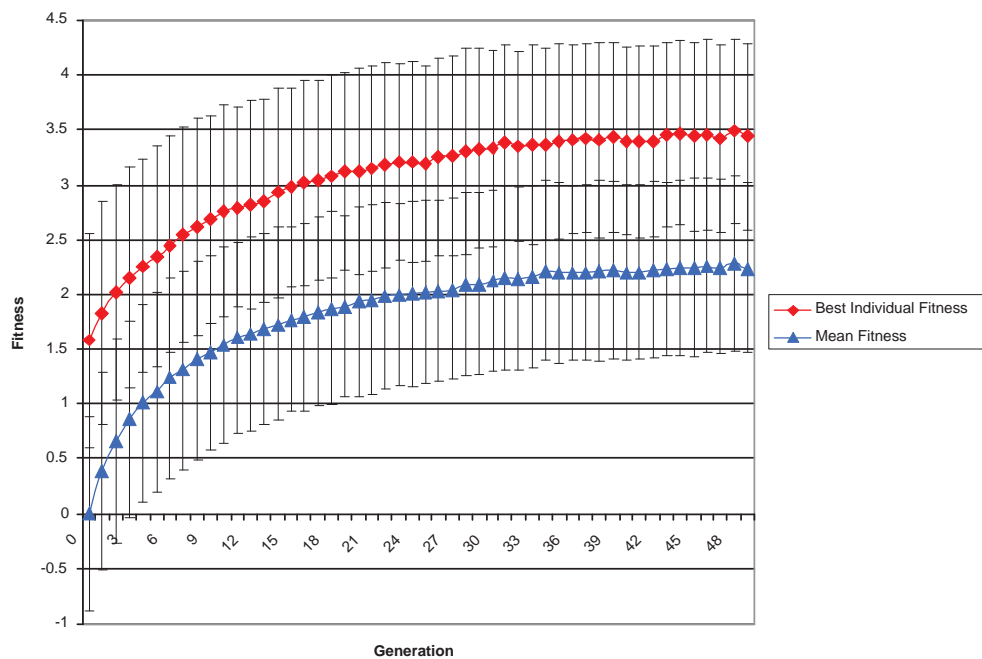


Figure 5: Evolution against random force

tection (P), detection (D), communication (C) and sustainment (S) attributes of the agent  $x \in \mathcal{A}$ . Let  $\mathcal{A}_y$  denote the random variable which represents the outcome of an agent's  $y$  attribute value at the completion of a simulation, where the agent is selected at random from the  $AT = 10 \times 10$  candidate agents. Then the Shannon entropy is

$$H(\mathcal{A}) = - \sum_{i \in \mathcal{A} | p(i) > 0} p(i) \log_b p(i), \quad (10)$$

where  $p(i)$  denotes the probability of the occurrence  $i \in \mathcal{A}$ .

The mean attribute values across all  $A$  agents in the  $T$  combined arms teams at the completion of each of the  $R$  simulations is presented in Table 1 together with the standard deviation for the mean and the base- $e$  Shannon entropy of the attributes over the  $ATR = 10 \times 10 \times 500$  samples.

Table 1: Mean attribute values

<i>Attribute</i>	<i>Mean</i>	<i>St.Dev.</i>	<i>Entropy</i>
Lethality (L)	3.379	0.275	1.633
Mobility (M)	2.266	0.231	1.488
Protection (P)	2.873	0.327	1.681
Detection (D)	3.403	0.263	1.609
Communication (C)	2.893	0.283	1.594
Sustainment (S)	3.185	0.281	1.592

The mean value for the attributes  $\bar{A}_y$ ,  $y \in \{L, M, P, D, C, S\}$ , in Table 1 can be interpreted only as the average total investment, in points per attribute, of a combined arms team per agent. The mean in no way denotes a representative agent for the combined arms team. The Shannon entropy measure is a better measure of the volatility in agents' attributes than the standard deviation because the standard deviation for the mean becomes arbitrarily small when sufficiently large numbers of simulations are conducted. For the sake of comparison, the uniform distribution over the Integers between one and ten has a Shannon Entropy of approximately 2.303 units and any unit point distribution has a Shannon Entropy of 0 units exactly.

The results of Table 1 are surprising in light of the analysis of Section 4.1. In particular, there is an unexpectedly high average total investment in detection at the cost of protection. One surmises that, in combined arms teams where agents evolve to counteract any deficiency held by the team as a whole and diversify to develop specialities, detection is the most universally important attribute while protection is only more important than mobility. However, the high variability in the mean for protection in relation to the other attributes indicates a greater range of values for that attribute. Agents rely on high values in other attributes and diverse combinations of agents with special functions within the team to offset lower survivability.

Finally, a blue force is evolved against a semi-balanced red force. This red force consists of agents initialised as balanced with attribute values  $P/6$  and then augmented twice, in a similar manner as the agents in Section 4.1; That is, by adding 1 unit to two attributes chosen at random. However, in this semi-balanced force the agents are independently generated. Hence, the red force is not composed of identical agents as are the forces

in Section 4.1. The average fitness of the genetic algorithm upon termination of the  $R$  simulations is approximately 1.12 with a standard deviation of 0.53 for the mean across all  $T$  combined arms teams and 2.50 with a standard deviation of 0.62 for the best combined arms team. This result is interesting in that agents in the blue force have  $P = 18$  attribute points total and agents in the semi-balanced red force have  $P + 2 = 21$  attribute points, approximately a 17% increase in capability over the blue force. This illustrates the very real value in diversity in the blue force and the importance of the right composition of agents.

### 4.3 Coevolved Strategies

A discussion of diversification and specialisation in a combined arms team is introduced in Section 4.2. In that section, combined arms teams are evolved against static enemy forces. In this section, we coevolve two combined arms teams against each other according to the two-population genetic algorithm of Section 3.4. A single simulation is conducted with input parameters as provided in Appendix A. A subset of the resultant strategies, which represent alternative combined arms teams, are analysed using a two-person zero-sum normal-form game, see Appendix C.

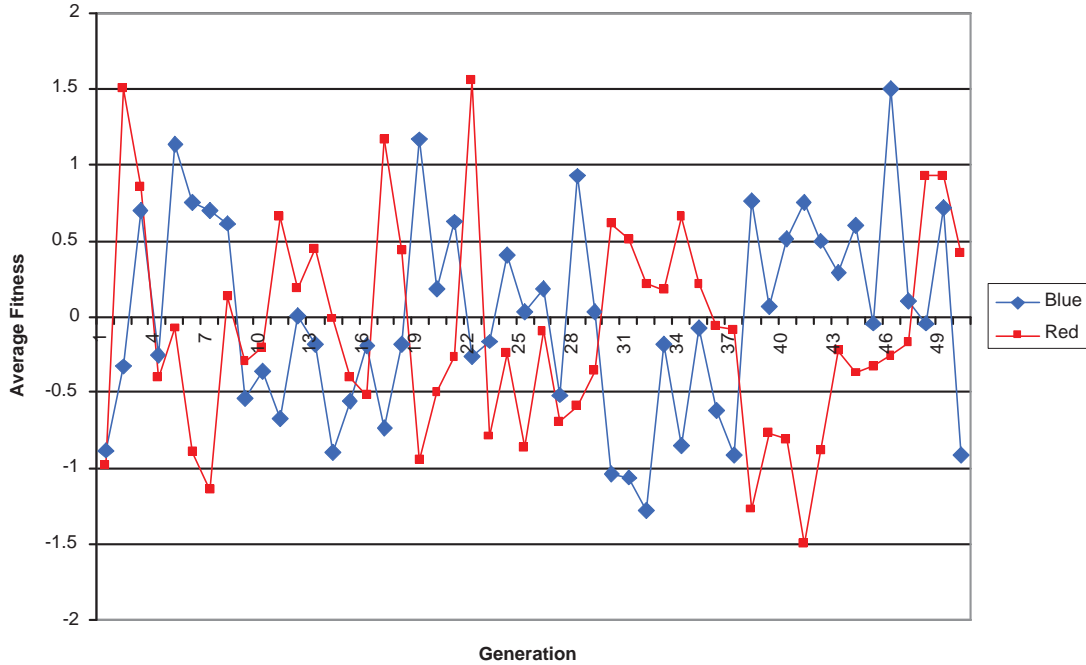


Figure 6: Coevolved average fitness

In Figure 6, the average fitness of the blue and red populations is displayed for a single simulation of the two-population genetic algorithm. The average fitness is calculated using  $R = 500$  independent calculations for the fitness, where each calculation simulates  $B = 20$  independent battles, of each combined arms team in each population against their opposing force's elected best combined arms team according to Section 3.4. The standard



deviations at all points can not be displayed using error bars because the largest deviation is approximately 0.003 which is too small to depict.

In Figure 6, it is interesting to note the interplay between the red and blue populations. As a trend, the two data sets seem to oscillate about the  $x$ -axis and each force opposes or counter-balances the efforts of the other. For example, near iteration 30 the red force has exploited a weakness in the blue force but near iteration 36 the blue force adapts to the red force and then dominates the battlefield until iteration 47.

To examine the actual composition of the combined arms teams resulting from coevolution, the two-person normal-form game  $\mathcal{G}$  is constructed as described in Appendix C. The red and blue populations at the end of three independent coevolution processes are recorded. These teams are displayed in Table 2 as six ordered sets of ten vectors  $(A_L(x), A_M(x), A_P(x), A_D(x), A_C(x), A_S(x))$ , for agents  $x$  in each respective combined arms team.

Table 2: Combined arms teams

Team	Agents				
1	(1,2,6,3,4,2)	(2,4,5,3,3,1)	(3,1,3,4,4,3)	(3,1,3,4,5,2)	(3,2,1,3,4,5)
	(3,3,2,3,3,4)	(3,4,4,2,3,2)	(4,1,1,4,4,4)	(4,1,3,3,3,4)	(5,2,2,3,2,4)
2	(1,1,3,3,5,5)	(1,1,6,5,1,4)	(2,1,6,2,2,5)	(2,2,4,4,4,2)	(3,2,4,2,4,3)
	(3,3,2,4,3,3)	(4,2,4,1,4,3)	(4,3,3,3,2,3)	(4,4,2,3,2,3)	(5,2,5,2,1,3)
3	(2,1,2,3,5,5)	(2,1,3,6,4,2)	(2,1,6,2,2,5)	(3,1,6,3,1,4)	(3,3,1,5,4,2)
	(3,3,3,4,2,3)	(4,2,3,4,2,3)	(5,2,3,3,4,1)	(6,2,3,3,2,2)	(8,1,3,1,1,4)
4	(2,1,4,3,5,3)	(2,3,2,3,4,4)	(3,1,1,4,3,6)	(3,3,1,4,4,3)	(3,3,1,5,4,2)
	(4,1,1,2,5,5)	(4,1,3,1,4,5)	(4,2,4,3,2,3)	(5,2,3,4,2,2)	(6,1,2,2,2,5)
5	(2,1,4,6,3,2)	(2,2,3,6,2,3)	(3,1,2,4,3,5)	(3,1,3,6,2,3)	(3,1,5,2,2,5)
	(3,1,5,3,4,2)	(3,2,5,4,1,3)	(3,3,2,4,2,4)	(4,3,2,4,1,4)	(6,1,3,1,4,3)
6	(1,1,4,8,3,1)	(1,2,2,2,6,5)	(2,1,4,4,4,3)	(2,2,4,3,1,6)	(3,1,3,6,3,2)
	(3,2,4,3,4,2)	(3,3,4,4,1,3)	(4,2,2,4,2,4)	(4,3,2,4,2,3)	(5,1,4,3,2,3)

All six strategies for the combined arms teams of Table 2 are available to each player irrespective of whether or not they originated as a red or blue strategy. If a strategy for a combined arms team is a good strategy for the blue force then it is an equally good strategy for the red team and *vice versa*. No constraints are imposed on either player as to which of the six strategies to employ.

To populate the normal-form matrix  $M$ , we define the payoff function  $P(i, j)$ ,  $i, j \in \mathcal{S}$ , as the average fitness  $\bar{f}_{i,j}$ , attained over  $R = 500$  independent fitness calculations, where each fitness calculation represents the average outcome of  $B = 20$  battle simulations conducted according to Section 2.3, and where the fitness of a single simulation is defined by equation (4). These values are presented to four decimal places in equation (11) as the zero-sum normal-form matrix  $M$ , where the leading diagonal is set to 0 and  $m_{i,j} = -m_{j,i}$  is enforced. The 99.9% confidence intervals for the entries  $m_{i,j}$  using a  $t$ -statistic with  $R - 1$  degrees of freedom are strictly less than 0.1. Hence, the confidence intervals for the means are quite small.

$$M = \begin{bmatrix} 0.0000 & 0.9750 & 0.3745 & 0.1845 & -0.7045 & -0.7780 \\ -0.9750 & 0.0000 & -1.9225 & -3.6380 & -6.0105 & -7.3375 \\ -0.3745 & 1.9225 & 0.0000 & 0.4340 & 0.1555 & 0.6465 \\ -0.1845 & 3.6380 & -0.4340 & 0.0000 & -0.8630 & -0.7605 \\ 0.7045 & 6.0105 & -0.1555 & 0.8630 & 0.0000 & 0.9525 \\ 0.7780 & 7.3375 & -0.6465 & 0.7605 & -0.9525 & 0.0000 \end{bmatrix}. \quad (11)$$

The reasons why certain combined arms teams defeat others are not easily identified. Looking at Table 2, one could not *a priori* predict the complex interactions depicted in equation (11). However, analysing the game  $\mathcal{G}$  helps us to understand the intrinsic value of combined arms teams.

Strategies two and four in equation (11) are strictly dominated and the matrix  $M$  reduces to

$$\hat{M} = \begin{bmatrix} 0.0000 & 0.3745 & -0.7045 & -0.7780 \\ -0.3745 & 0.0000 & 0.1555 & 0.6465 \\ 0.7045 & -0.1555 & 0.0000 & 0.9525 \\ 0.7780 & -0.6465 & -0.9525 & 0.0000 \end{bmatrix}. \quad (12)$$

In  $\hat{M}$ , there is no “best” combined arms team because the  $\hat{M}$  contains no saddle point solution. The combined arms teams interact with each other to defy transitivity. Team 1 defeats team 3 and team 3 defeats team 5. However, team 5 also defeats team 1. Figure 7 depicts this behaviour.

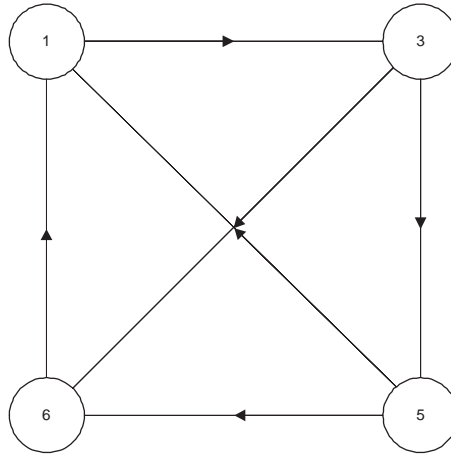


Figure 7: Defeats network

The behaviour displayed in Figure 7 has potentially significant implications for combined arms teams. The analysis demonstrates that there may be no best force in general and that the force may need to be able to adapt itself in order to defeat different opponents. Thus, future concepts for combined arms teaming should emphasise the development of agile and rapidly re-configurable forces. This section implicitly assumes both sides of a conflict can co-adapt to each other with equal ease. This need not necessarily be the case. In reality, the rate at which a force can adapt to the opposition is likely to have significant impact on the outcome of conflict. Hence, value in capability comes not just from the

number of different capability options one has at hand but also with the rate at which different options can be utilised.

The zero-sum normal-form game  $\hat{\mathcal{G}}$  with payoff matrix  $\hat{M}$  has the solution  $\hat{S}$  of all linear convex combinations of

$$\hat{S} = \left\{ \begin{array}{l} (0.3594, \quad 0.4325, \quad 0.0000, \quad 0.2082), \\ (0.1260, \quad 0.5707, \quad 0.3034, \quad 0.0000) \end{array} \right\}, \quad (13)$$

which correspond to the linear convex combinations of

$$S = \left\{ \begin{array}{l} (0.3594, \quad 0.0000, \quad 0.4325, \quad 0.0000, \quad 0.0000, \quad 0.2082), \\ (0.1260, \quad 0.0000, \quad 0.5707, \quad 0.0000, \quad 0.3034, \quad 0.0000) \end{array} \right\}, \quad (14)$$

in the original game  $\mathcal{G}$ .

The set of six combined arms teams given in Table 2 is constructed almost arbitrarily for the sake of example. We do not envisage candidate combined arms teams be generated according to a genetic algorithm in non-theoretical studies. In reality, various options for combined arms teams might be generated to meet a number of physical constraints based on resources, existing capabilities or future acquisitions. However, once a set of candidate combined arms teams is constructed, games theory provides a number of useful insights and techniques of analysis.

For any zero-sum normal-form, one can explore the consequences for either player of losing one or more strategies from their respective sets of strategies. Loss of a strategy has the potential to influence the value of the game. Each player can at best maintain the value of the game and is placed at a disadvantage whenever the strategies lost are a part of a mixed strategy solution of the game. In our example, teams 2 and 4 are unnecessary because they are not used in any solution of the game. Only one of teams 5 and 6 are needed to guarantee the value of the game so that if one of the teams 5 and 6 are removed from either player's strategies then that player can maintain the value of the game. On the other hand, teams 1 and 3 are essential as both of these teams appear in every mixed strategy solution of the game. When the combined arms team 1 is removed from player 1's set of strategies the value of the game decreases by 0.0472 and when the combined arms team 3 is removed from player 1's set of strategies the value of the game decreases by 0.0887.

The value of the game is a useful way to determine the relative benefits and costs of various sets of combined arms teams. Options for combined arms teams can be added and removed and the impact on the value of the game against a baseline can be measured. However, there are some limitations to this approach. For example, suppose that the payoffs for some strategy  $\hat{s}$  approach that of strategy  $s$  from below such that strategy  $\hat{s}$  is dominated by strategy  $s$  by an arbitrarily small amount  $\epsilon > 0$ . Then, removal of strategy  $s$  from player 1's options has little impact on the value of the game because strategy  $\hat{s}$  can be substituted for strategy  $s$  and decrease the value of the game by at most  $\epsilon$ . Suppose that strategy  $s$  is originally removed from player 1's options because the platforms or assets that comprise this strategy are destroyed. Then it may also be that strategy  $\hat{s}$ , or any other strategy, contains many of the same platforms as strategy  $s$  and thus cannot in all practicality act as a substitute for strategy  $s$ . The importance of strategies to each player therein lies both in the payoffs gained by employing those strategies and also in the

abstract notion of a strategy denoting a genuine reserve or back-up capability that can be fielded if required.

In many cases, it is reasonable to assess the impact on the value of the game of substituting, not entire combined arms teams, but individual assets of combined arms teams. Suppose that four distinct combined arms teams appear in the optimal strategy of the game and that an agent representing a 120 mm mortar appears in only one of these strategies. It may be costly to maintain a 120 mm mortar capability simply for one potential combined arms team. Thus, we may wish to replace that capability by say a 105 mm artillery platform and then judge whether or not the decrease (or increase) in the value of the game is acceptable when weighed against the perceived cost of fielding a 120 mm mortar system. Likewise, the relative merits of integrating a number of different weapons systems into a given combined arms team can be measured.

## 5 Discussion

In this study it is difficult to interpret the agents' attributes in terms of physical systems. To address this problem, agents' attributes could be implemented to represent real-life capabilities and the model scaled appropriately. For example, an attribute of 1 for lethality could be set to denote small-arms, 2 to denote a 81 mm artillery using standard HE point detonating rounds, 3 to denote a ground-based air-defence capability similar to a shoulder launched ground-to-air missile, and so on. With this approach, restrictions on the use of weapons, realistic weapons effects, realistic Intelligence, Surveillance and Reconnaissance (ISR), and range-based and line-of-sight restricted communications can be incorporated into the model. Of course, this also requires that terrain, vegetation and elevation be added to the battlefield. The refinement of detection into detection, recognition and identification for ISR based on this new terrain provides the potential to study fratricide and to explore the agent behaviours leading to it.

Currently, the behaviours of agents in battles are fixed. Agents, irrespective of their attributes, employ the same movement, engagement, detection, communication and sustainment rules. Potentially, adaptive learning can be introduced into the model to enable agents to choose from a set of candidate behaviours and then learn from those choices, see Baker et al. (2004). Agents then adopt various specialised roles or functions according to their attributes to accomplish simple tasks, such as taking and holding the centre of the battlefield. This approach has the potential to implement and use sophisticated tactics and greatly enhances the usefulness and realism of the model. It is also possible that games theory solutions be used within the combat model, genetic algorithm and learning algorithms. Such a hybrid approach facilitates a more practical valuation of the combined arms teams and the agents within those teams than does the simple fitness value based on casualties.

Future applications of this research include a study the impact of parameters other than agents' attributes and behaviours. For example, a sensitivity analysis of combined arms teams sizes and the investment of resources across the teams is to be conducted. In such a study, force commanders select the number of agents, to deploy and allocate attribute values across those agents up to a fixed amount. Such a study provides insights into the

benefits and limitations of deploying large combined arms teams against smaller teams and the trade off between numerical superiority and enhanced capability. Other potential studies include measuring the impact that the scenario instantiation itself has on the outcome of combat. This includes variations in the model to better capture operational environment and operational tempo. Such modifications can be made even without the implementation of a sophisticated terrain model but are more difficult to calibrate and interpret without one. The parameters which define these scenarios then have a follow on benefit to agents' attributes and the composition of combined arms teams. For example, the requirement for an enhanced detection capability may prove more important in complex physical terrain than open ground.

Finally, extensions to this study include introducing a number  $K$  of different forces (including non-combatants) into the battlefield, each with tendencies, behaviours and objectives as well as Command and Control structures modelled using embedded social networks. A cooperative game emerges as the  $K$  factions ally to accomplish set goals.

## 6 Conclusions

This study has presented a foundation for the analysis of the costs, benefits and limitations inherent to combined arms teaming in a simulated environment. An agent-based approach was developed for a generic combat model used to simulate combined arms teams. In this model, six key attributes of combined arms teams were studied: lethality; mobility; protection; detection; communication; and sustainment. A limited parametric study was conducted to explore how the attributes interacted and the simulation results analysed using statistical techniques to guarantee reasonable confidence in the data collected. These results were tested for consistency using the Analytic Hierarchy Process and subsequently used to determine the relative importance or priority for the combined arms teams in the parametric study. It was broadly concluded that lethality, sustainment and protection had the greatest influence on the outcome of combat, with detection and communication having a lesser impact. The usefulness or benefit imparted by mobility could not be determined in this study.

The parametric study, mentioned above, was conducted by assuming all members of a combined arms teams were identical. To explore the benefits of diversity and specialisation within combined arms teams, a genetic algorithm was employed. This genetic algorithm evolved the combined arms teams against an opposing force whose combined arms teams were kept constant. The results of this evolution process were consistent with the parametric study with one notable exception. It was found that the total investment in detection across all members of the combined arms teams was unexpectedly high while the total investment in protection unexpectedly low. We inferred that, in combined arms teams where agents evolve to counteract any deficiency held by the team as a whole and diversify to take on special roles and functions within the combined arms teams, detection was the most universally important attribute. It was also demonstrated that combined arms teams evolved using the genetic algorithm could defeat combined arms teams even when those teams were superior in capability.

Finally, two opposing forces were coevolved against each other using a two-population

genetic algorithm. A games theoretic analysis of combined arms teams using a two-person zero-sum normal-form game was conducted. This analysis was used to demonstrate techniques to study the impact, costs, and benefits of various strategies for combined arms teams. The value of the game and the solution of the game were used to provide a quantitative valuation of the consequences of adding, removing and replacing capabilities or assets within combined arms teams. Our analysis indicates that there may not be a best combined arms team for the Land force. Combined arms teams may need to be self-adapting to defeat a range of different opponents. Thus, the potential benefits of agile and rapidly re-configurable forces are demonstrated, as are the potential limitations of static forces, inclusive of static forces optimised against known criteria. Value in capability then comes from both the number of alternative capability options a force possesses and also the rate at which those options can be utilised.

The techniques employed in this study were sufficiently general to be adapted to a number of alternative approaches other than agent-based simulation. For example, the Analytic Hierarchy Process is typically used to study data collected from subject matter experts. We have used it to analyse data obtained from computer simulation. However, the two approaches are not mutually exclusive. A combat simulator is not necessary in order to gain insights into combined arms teaming. An independent study using subject matter advice and military seminar wargames can be used in conjunction with simulation. Future studies using these approaches have the potential to provide valuable insights into the relative benefits of combined arms teams and to provide advice on the composition of these teams for the Land force.

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## Appendix A: Input Parameters

Table A1: Input parameters

<i>Parameter</i>	<i>Value</i>	<i>Meaning</i>
$P$	18	Sum of agents' attribute values.
$A$	50	Maximum number of times agents may act.
$N$	10	Number of agents in a team.
$T$	5	Number of teams in a population.
$G$	50	Number of generations.
$B$	20	Number of independent battles conducted to calculate the average fitness of a given blue and red team.
$R$	500	Number of independent replications used for statistics.
$x_{OBJ}$	50	$x$ coordinate value for the mission objective point.
$y_{OBJ}$	50	$y$ coordinate value for the mission objective point.
$p_{CROSS}$	0.6	Probability of crossover.
$p_{MUTATE}$	0.2	Probability of mutation.

## Appendix B: Simulation Data

Table B1: Simulation results

<b>Id No.</b>	<b>Mean</b>	<b>Variance</b>	<b>Interval +/-</b>	<b>Matchup: Force 1 against Force 2</b>					
0	2.3088	0.3187	0.0836	leth.	vs	mob.			
1	-0.4458	0.3835	0.0917	leth.	vs	prot.			
2	1.5795	0.3578	0.0886	leth.	vs	det.			
3	0.7107	0.3590	0.0887	leth.	vs	com.			
4	-0.0133	0.3853	0.0919	leth.	vs	sust.			
5	0.9951	0.3132	0.0828	leth.	vs	leth.	+	mob.	
6	-0.3766	0.3387	0.0861	leth.	vs	leth.	+	prot.	
7	0.1924	0.4489	0.0992	leth.	vs	leth.	+	det.	
8	-0.5184	0.3708	0.0901	leth.	vs	leth.	+	com.	
9	-0.5653	0.3609	0.0889	leth.	vs	leth.	+	sust.	
10	1.1125	0.3667	0.0896	leth.	vs	mob.	+	prot.	
11	1.6200	0.3423	0.0866	leth.	vs	mob.	+	det.	
12	1.5721	0.3393	0.0862	leth.	vs	mob.	+	com.	
13	1.2116	0.3078	0.0821	leth.	vs	mob.	+	sust.	
14	0.4908	0.3999	0.0936	leth.	vs	prot.	+	det.	
15	-0.1841	0.3682	0.0898	leth.	vs	prot.	+	com.	
16	-0.2209	0.4164	0.0955	leth.	vs	prot.	+	sust.	
17	1.0795	0.3355	0.0857	leth.	vs	det.	+	com.	
18	0.4439	0.3771	0.0909	leth.	vs	det.	+	sust.	
19	0.3144	0.3581	0.0886	leth.	vs	com.	+	sust.	
20	-3.4060	0.3271	0.0847	mob.	vs	prot.			
21	-1.9346	0.3676	0.0898	mob.	vs	det.			
22	-1.7715	0.3445	0.0869	mob.	vs	com.			
23	-2.3816	0.3072	0.0820	mob.	vs	sust.			
24	-1.3562	0.4087	0.0946	mob.	vs	leth.	+	mob.	
25	-3.0954	0.3384	0.0861	mob.	vs	leth.	+	prot.	
26	-2.6301	0.3816	0.0914	mob.	vs	leth.	+	det.	
27	-2.4243	0.2952	0.0804	mob.	vs	leth.	+	com.	
28	-2.6469	0.2983	0.0808	mob.	vs	leth.	+	sust.	
29	-1.5395	0.4046	0.0942	mob.	vs	mob.	+	prot.	
30	-0.7513	0.4266	0.0967	mob.	vs	mob.	+	det.	
31	-0.8584	0.3429	0.0867	mob.	vs	mob.	+	com.	
32	-1.3312	0.3826	0.0916	mob.	vs	mob.	+	sust.	
33	-2.7655	0.3835	0.0917	mob.	vs	prot.	+	det.	
34	-2.5988	0.3547	0.0882	mob.	vs	prot.	+	com.	
35	-2.9248	0.3477	0.0873	mob.	vs	prot.	+	sust.	
36	-2.1566	0.3451	0.0870	mob.	vs	det.	+	com.	
37	-2.5367	0.3257	0.0845	mob.	vs	det.	+	sust.	
38	-2.1242	0.3067	0.0820	mob.	vs	com.	+	sust.	
39	2.4924	0.4468	0.0989	prot.	vs	det.			
40	1.4047	0.4173	0.0956	prot.	vs	com.			
41	-0.0224	0.4428	0.0985	prot.	vs	sust.			

Table B2: Simulation results

<b>Id No.</b>	<b>Mean</b>	<b>Variance</b>	<b>Interval +/-</b>	<b>Matchup: Force 1 against Force 2</b>							
42	1.5842	0.3974	0.0933	prot.	vs	leth.	+	mob.			
43	0.0878	0.4948	0.1041	prot.	vs	leth.	+	prot.			
44	1.1444	0.4066	0.0944	prot.	vs	leth.	+	det.			
45	0.6592	0.4233	0.0963	prot.	vs	leth.	+	com.			
46	-0.2225	0.3972	0.0933	prot.	vs	leth.	+	sust.			
47	1.7297	0.4119	0.0950	prot.	vs	mob.	+	prot.			
48	2.5452	0.4304	0.0971	prot.	vs	mob.	+	det.			
49	2.3300	0.3689	0.0899	prot.	vs	mob.	+	com.			
50	1.4123	0.3569	0.0884	prot.	vs	mob.	+	sust.			
51	0.4233	0.4377	0.0979	prot.	vs	prot.	+	det.			
52	-0.5197	0.4339	0.0975	prot.	vs	prot.	+	com.			
53	-0.1601	0.4372	0.0979	prot.	vs	prot.	+	sust.			
54	1.8908	0.4319	0.0973	prot.	vs	det.	+	com.			
55	0.9304	0.4570	0.1001	prot.	vs	det.	+	sust.			
56	0.4251	0.4667	0.1011	prot.	vs	com.	+	sust.			
57	-0.4082	0.4066	0.0944	det.	vs	com.					
58	-1.6212	0.3916	0.0926	det.	vs	sust.					
59	0.0331	0.3843	0.0918	det.	vs	leth.	+	mob.			
60	-2.1444	0.3756	0.0907	det.	vs	leth.	+	prot.			
61	-1.3102	0.4738	0.1019	det.	vs	leth.	+	det.			
62	-1.4479	0.3736	0.0905	det.	vs	leth.	+	com.			
63	-1.7636	0.3817	0.0915	det.	vs	leth.	+	sust.			
64	-0.4428	0.4776	0.1023	det.	vs	mob.	+	prot.			
65	0.4651	0.4020	0.0939	det.	vs	mob.	+	det.			
66	0.3627	0.3866	0.0920	det.	vs	mob.	+	com.			
67	0.0681	0.3890	0.0923	det.	vs	mob.	+	sust.			
68	-1.2141	0.4740	0.1019	det.	vs	prot.	+	det.			
69	-1.5493	0.4273	0.0968	det.	vs	prot.	+	com.			
70	-2.0554	0.4032	0.0940	det.	vs	prot.	+	sust.			
71	-0.4369	0.3899	0.0924	det.	vs	det.	+	com.			
72	-1.0955	0.4324	0.0973	det.	vs	det.	+	sust.			
73	-1.2711	0.3843	0.0918	det.	vs	com.	+	sust.			
74	-0.7951	0.3538	0.0881	com.	vs	sust.					
75	0.3714	0.3101	0.0824	com.	vs	leth.	+	mob.			
76	-1.2029	0.4209	0.0960	com.	vs	leth.	+	prot.			
77	-0.4852	0.4147	0.0953	com.	vs	leth.	+	det.			
78	-0.6612	0.3715	0.0902	com.	vs	leth.	+	com.			
79	-1.0007	0.3943	0.0930	com.	vs	leth.	+	sust.			
80	0.3603	0.3601	0.0888	com.	vs	mob.	+	prot.			
81	0.9705	0.3863	0.0920	com.	vs	mob.	+	det.			
82	0.9324	0.3230	0.0841	com.	vs	mob.	+	com.			
83	0.5909	0.3401	0.0863	com.	vs	mob.	+	sust.			

Table B3: Simulation results

<b>Id No.</b>	<b>Mean</b>	<b>Variance</b>	<b>Interval +/-</b>		<b>Matchup: Force 1 against Force 2</b>						
84	-0.6088	0.4454	0.0988	com.	vs	prot.	+	det.			
85	-0.5706	0.4103	0.0948	com.	vs	prot.	+	com.			
86	-1.1265	0.3883	0.0922	com.	vs	prot.	+	sust.			
87	0.3285	0.3762	0.0908	com.	vs	det.	+	com.			
88	-0.5334	0.3396	0.0863	com.	vs	det.	+	sust.			
89	-0.4531	0.3587	0.0887	com.	vs	com.	+	sust.			
90	1.0864	0.3737	0.0905	sust.	vs	leth.	+	mob.			
91	-0.2630	0.4018	0.0938	sust.	vs	leth.	+	prot.			
92	0.2324	0.3742	0.0906	sust.	vs	leth.	+	det.			
93	0.1384	0.3681	0.0898	sust.	vs	leth.	+	com.			
94	-0.2620	0.3649	0.0894	sust.	vs	leth.	+	sust.			
95	1.4410	0.3496	0.0875	sust.	vs	mob.	+	prot.			
96	1.7057	0.3416	0.0865	sust.	vs	mob.	+	det.			
97	1.7728	0.3668	0.0897	sust.	vs	mob.	+	com.			
98	1.3409	0.3226	0.0841	sust.	vs	mob.	+	sust.			
99	0.7689	0.4529	0.0996	sust.	vs	prot.	+	det.			
100	0.4806	0.3944	0.0930	sust.	vs	prot.	+	com.			
101	-0.0265	0.3815	0.0914	sust.	vs	prot.	+	sust.			
102	1.1456	0.3252	0.0844	sust.	vs	det.	+	com.			
103	0.4620	0.4811	0.1027	sust.	vs	det.	+	sust.			
104	0.3443	0.3356	0.0858	sust.	vs	com.	+	sust.			
105	-1.4954	0.3850	0.0919	leth.	+	mob.	vs	leth.	+	prot.	
106	-0.9281	0.3764	0.0908	leth.	+	mob.	vs	leth.	+	det.	
107	-1.0102	0.3377	0.0860	leth.	+	mob.	vs	leth.	+	com.	
108	-1.3610	0.3522	0.0879	leth.	+	mob.	vs	leth.	+	sust.	
109	-0.0789	0.4538	0.0997	leth.	+	mob.	vs	mob.	+	prot.	
110	0.7192	0.3706	0.0901	leth.	+	mob.	vs	mob.	+	det.	
111	0.5101	0.3981	0.0934	leth.	+	mob.	vs	mob.	+	com.	
112	0.0511	0.4144	0.0953	leth.	+	mob.	vs	mob.	+	sust.	
113	-0.7662	0.3808	0.0913	leth.	+	mob.	vs	prot.	+	det.	
114	-0.9296	0.3590	0.0887	leth.	+	mob.	vs	prot.	+	com.	
115	-1.3533	0.3557	0.0883	leth.	+	mob.	vs	prot.	+	sust.	
116	-0.3186	0.3994	0.0936	leth.	+	mob.	vs	det.	+	com.	
117	-0.8249	0.3307	0.0851	leth.	+	mob.	vs	det.	+	sust.	
118	-0.7959	0.3185	0.0835	leth.	+	mob.	vs	com.	+	sust.	
119	0.8321	0.4110	0.0949	leth.	+	prot.	vs	leth.	+	det.	
120	0.5291	0.4034	0.0940	leth.	+	prot.	vs	leth.	+	com.	
121	-0.0522	0.3061	0.0819	leth.	+	prot.	vs	leth.	+	sust.	
122	1.5925	0.4398	0.0982	leth.	+	prot.	vs	mob.	+	prot.	
123	2.2844	0.3596	0.0888	leth.	+	prot.	vs	mob.	+	det.	
124	2.0743	0.3439	0.0868	leth.	+	prot.	vs	mob.	+	com.	
125	1.5837	0.3447	0.0869	leth.	+	prot.	vs	mob.	+	sust.	

Table B4: Simulation results

<b>Id No.</b>	<b>Mean</b>	<b>Variance</b>	<b>Interval +/-</b>	<b>Matchup: Force 1 against Force 2</b>						
126	1.1535	0.3811	0.0914	leth.	+	prot.	vs	prot.	+	det.
127	0.6675	0.4347	0.0976	leth.	+	prot.	vs	prot.	+	com.
128	0.0202	0.3997	0.0936	leth.	+	prot.	vs	prot.	+	sust.
129	1.6395	0.3873	0.0921	leth.	+	prot.	vs	det.	+	com.
130	0.8656	0.4302	0.0971	leth.	+	prot.	vs	det.	+	sust.
131	0.5443	0.3818	0.0915	leth.	+	prot.	vs	com.	+	sust.
132	-0.0392	0.3915	0.0926	leth.	+	det.	vs	leth.	+	com.
133	-0.4404	0.3926	0.0928	leth.	+	det.	vs	leth.	+	sust.
134	0.7068	0.4096	0.0947	leth.	+	det.	vs	mob.	+	prot.
135	1.5198	0.3958	0.0931	leth.	+	det.	vs	mob.	+	det.
136	1.4236	0.4127	0.0951	leth.	+	det.	vs	mob.	+	com.
137	1.1651	0.4105	0.0948	leth.	+	det.	vs	mob.	+	sust.
138	0.1856	0.4287	0.0969	leth.	+	det.	vs	prot.	+	det.
139	-0.1251	0.4488	0.0992	leth.	+	det.	vs	prot.	+	com.
140	-0.6470	0.4306	0.0971	leth.	+	det.	vs	prot.	+	sust.
141	0.8824	0.4294	0.0970	leth.	+	det.	vs	det.	+	com.
142	0.3056	0.4912	0.1037	leth.	+	det.	vs	det.	+	sust.
143	0.1127	0.4782	0.1024	leth.	+	det.	vs	com.	+	sust.
144	-0.3779	0.3338	0.0855	leth.	+	com.	vs	leth.	+	sust.
145	1.0698	0.3743	0.0906	leth.	+	com.	vs	mob.	+	prot.
146	1.6081	0.3506	0.0876	leth.	+	com.	vs	mob.	+	det.
147	1.5628	0.3377	0.0860	leth.	+	com.	vs	mob.	+	com.
148	1.2136	0.3343	0.0856	leth.	+	com.	vs	mob.	+	sust.
149	0.4703	0.4272	0.0968	leth.	+	com.	vs	prot.	+	det.
150	0.1382	0.4213	0.0961	leth.	+	com.	vs	prot.	+	com.
151	-0.3522	0.3977	0.0933	leth.	+	com.	vs	prot.	+	sust.
152	0.9590	0.3551	0.0882	leth.	+	com.	vs	det.	+	com.
153	0.3437	0.3676	0.0898	leth.	+	com.	vs	det.	+	sust.
154	0.1992	0.3436	0.0868	leth.	+	com.	vs	com.	+	sust.
155	1.6985	0.3709	0.0902	leth.	+	sust.	vs	mob.	+	prot.
156	1.9524	0.3784	0.0911	leth.	+	sust.	vs	mob.	+	det.
157	1.8585	0.3226	0.0841	leth.	+	sust.	vs	mob.	+	com.
158	1.5829	0.3351	0.0857	leth.	+	sust.	vs	mob.	+	sust.
159	1.0433	0.4143	0.0953	leth.	+	sust.	vs	prot.	+	det.
160	0.8021	0.4233	0.0963	leth.	+	sust.	vs	prot.	+	com.
161	-0.0906	0.4187	0.0958	leth.	+	sust.	vs	prot.	+	sust.
162	1.3217	0.3546	0.0881	leth.	+	sust.	vs	det.	+	com.
163	0.7215	0.3623	0.0891	leth.	+	sust.	vs	det.	+	sust.
164	0.6177	0.3659	0.0895	leth.	+	sust.	vs	com.	+	sust.
165	1.0540	0.4428	0.0985	mob.	+	prot.	vs	mob.	+	det.
166	0.6370	0.3945	0.0930	mob.	+	prot.	vs	mob.	+	com.
167	-0.0253	0.3583	0.0886	mob.	+	prot.	vs	mob.	+	sust.

Table B5: Simulation results

<b>Id No.</b>	<b>Mean</b>	<b>Variance</b>	<b>Interval +/-</b>	<b>Matchup: Force 1 against Force 2</b>						
168	-0.7246	0.4676	0.1012	mob.	+	prot.	vs	prot.	+	det.
169	-1.0847	0.4301	0.0971	mob.	+	prot.	vs	prot.	+	com.
170	-1.7686	0.3882	0.0922	mob.	+	prot.	vs	prot.	+	sust.
171	-0.1888	0.4236	0.0963	mob.	+	prot.	vs	det.	+	com.
172	-1.0322	0.4176	0.0957	mob.	+	prot.	vs	det.	+	sust.
173	-1.1649	0.3762	0.0908	mob.	+	prot.	vs	com.	+	sust.
174	-0.2562	0.4276	0.0968	mob.	+	det.	vs	mob.	+	com.
175	-0.6825	0.3980	0.0934	mob.	+	det.	vs	mob.	+	sust.
176	-1.5784	0.3928	0.0928	mob.	+	det.	vs	prot.	+	det.
177	-1.7564	0.3907	0.0925	mob.	+	det.	vs	prot.	+	com.
178	-2.1518	0.3631	0.0892	mob.	+	det.	vs	prot.	+	sust.
179	-0.9189	0.3736	0.0905	mob.	+	det.	vs	det.	+	com.
180	-1.4625	0.3923	0.0927	mob.	+	det.	vs	det.	+	sust.
181	-1.3669	0.3937	0.0929	mob.	+	det.	vs	com.	+	sust.
182	-0.4764	0.3891	0.0923	mob.	+	com.	vs	mob.	+	sust.
183	-1.3636	0.4131	0.0951	mob.	+	com.	vs	prot.	+	det.
184	-1.5868	0.3835	0.0917	mob.	+	com.	vs	prot.	+	com.
185	-2.0546	0.3360	0.0858	mob.	+	com.	vs	prot.	+	sust.
186	-0.8722	0.3969	0.0933	mob.	+	com.	vs	det.	+	com.
187	-1.4758	0.3797	0.0912	mob.	+	com.	vs	det.	+	sust.
188	-1.3811	0.3488	0.0874	mob.	+	com.	vs	com.	+	sust.
189	-0.7867	0.4003	0.0937	mob.	+	sust.	vs	prot.	+	det.
190	-1.0006	0.3616	0.0890	mob.	+	sust.	vs	prot.	+	com.
191	-1.4162	0.3660	0.0896	mob.	+	sust.	vs	prot.	+	sust.
192	-0.5645	0.3895	0.0924	mob.	+	sust.	vs	det.	+	com.
193	-1.0879	0.3459	0.0871	mob.	+	sust.	vs	det.	+	sust.
194	-1.0671	0.3759	0.0908	mob.	+	sust.	vs	com.	+	sust.
195	-0.4444	0.4344	0.0976	prot.	+	det.	vs	prot.	+	com.
196	-1.1507	0.3999	0.0936	prot.	+	det.	vs	prot.	+	sust.
197	0.6919	0.4357	0.0977	prot.	+	det.	vs	det.	+	com.
198	-0.0875	0.5003	0.1047	prot.	+	det.	vs	det.	+	sust.
199	-0.3183	0.4264	0.0967	prot.	+	det.	vs	com.	+	sust.
200	-0.6819	0.4404	0.0982	prot.	+	com.	vs	prot.	+	sust.
201	1.0706	0.4525	0.0996	prot.	+	com.	vs	det.	+	com.
202	0.2257	0.4117	0.0950	prot.	+	com.	vs	det.	+	sust.
203	-0.0582	0.3689	0.0899	prot.	+	com.	vs	com.	+	sust.
204	1.5068	0.4019	0.0939	prot.	+	sust.	vs	det.	+	com.
205	0.7104	0.4624	0.1007	prot.	+	sust.	vs	det.	+	sust.
206	0.4056	0.4119	0.0950	prot.	+	sust.	vs	com.	+	sust.
207	-0.5760	0.4712	0.1016	det.	+	com.	vs	det.	+	sust.
208	-0.6905	0.3930	0.0928	det.	+	com.	vs	com.	+	sust.
209	-0.1288	0.3858	0.0919	det.	+	sust.	vs	com.	+	sust.

## Appendix C: Games Theory

### A.1 Two-Person Zero-Sum Games

Before we formally define what is meant by a game, we first introduce the components that make up such a game. Let  $\mathcal{P} = \{1, 2, \dots, N\}$  denote the finite set of players participating in the game. Let  $\mathcal{S}_i$  denote the sets of strategies available to the respective players  $i \in \mathcal{P}$ . Then, the set of all possible outcomes of the game is  $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_1 \times \dots \times \mathcal{S}_N$ . Let  $U_i : \mathcal{S} \rightarrow \mathbb{R}$  denote the payoff function which defines the benefit received by player  $i \in \mathcal{P}$  as a result of the game. Then, the  $N$ -person normal-form game is defined by the set of players  $\mathcal{P}$  partaking of the game, the sets of strategies  $\mathcal{S}_i$ ,  $i = 1 \dots N$ , available to the players and the payoff functions  $U_i$ ,  $i = 1 \dots N$ .

A two-person normal-form game (Dresher, 1961; Isaacs, 1965) with a finite set of strategies is representable as a two dimensional payoff matrix  $M$  of size  $|\mathcal{S}_1| \times |\mathcal{S}_2|$ . Hence, this matrix has a row for each strategy  $i \in \mathcal{S}_1$  and a column for each strategy  $j \in \mathcal{S}_2$ . The  $(i, j)^{\text{th}}$  entries of  $M$  contain the payoff tuples  $(u, v) = (U_1(i), U_2(j))$  corresponding to the payoffs  $u$  and  $v$  received by player 1 and player 2 respectively upon playing strategy  $i \in \mathcal{S}_1$  and strategy  $j \in \mathcal{S}_2$  respectively.

For example, the game with two players having  $m$  and  $n$  strategies respectively, and a payoff function  $U(i, j) = (u_{i,j}, v_{i,j})$ ,  $i = 1 \dots m$ ,  $j = 1 \dots n$ , has

$$M = \begin{bmatrix} (u_{1,1}, v_{1,1}) & (u_{1,2}, v_{1,2}) & \dots & (u_{1,n}, v_{1,n}) \\ (u_{2,1}, v_{2,1}) & (u_{2,2}, v_{2,2}) & \dots & (u_{2,n}, v_{2,n}) \\ \vdots & \vdots & \ddots & \vdots \\ (u_{m,1}, v_{m,1}) & (u_{m,2}, v_{m,2}) & \dots & (u_{m,n}, v_{m,n}) \end{bmatrix}. \quad (15)$$

A zero-sum two-person normal-form game with a finite number of strategies is a special class of game in which  $U_1(i, j) = -U_2(i, j)$ ,  $\forall i \in \mathcal{S}_1, j \in \mathcal{S}_2$ . Hence, neither player benefits from collaboration with their opponent. For such games it is convenient to define the payoff function  $P \equiv U_1$ , the set of strategies for player 1 as  $X \equiv \mathcal{S}_1$  and the set of strategies for player 2 as  $Y \equiv \mathcal{S}_2$ . By convention, the normal-form matrix for these games contains only the entries  $P(i, j)$ ,  $i \in X$ ,  $j \in Y$ .

In this paper, we will consider only zero-sum two-person normal-form games  $\mathcal{G} = (X, Y, P)$  of complete-information with a finite number of strategies. In these games, each player is assumed to know  $X$ ,  $Y$  and  $U$ . Furthermore, outcome of the game is resolved simultaneously for both players such that neither player is advantaged by knowing *a priori* the strategy played by the opponent.

### A.2 Dominated Strategies

If there exists co-efficients  $k_i \geq 0$ , such that  $P(x, j) \leq \sum_{i \neq x} k_i P(i, j)$ , for all  $j \in Y$ , and  $\sum_i k_i = 1$ , then we say that  $x$  is a dominated strategy of the game. Similarly, if there exists co-efficients  $l_j \geq 0$ , such that  $P(i, y) \leq \sum_{j \neq y} l_j P(i, j)$ , for all  $j \in Y$ , and

$\sum_j l_j = 1$ , then we say that  $y$  is a dominated strategy of the game, see the classical text Luce and Raiffa (1957) for the original explanation dominated strategies. Dominated strategies are removed from the game because there exist convex linear combinations of alternative strategies with the same or superior payoffs. A sub-game arises as the result of this process. Furthermore, the process is then applied to this sub-game, if required. The process is repeated as many times as necessary and is referred to as iterated elimination of dominated strategies.

### A.3 Saddle Points

We now wish to determine the optimal playing strategies for the two players in a two person game, see von Neumann (1928) for an original analysis and derivation of minimax solutions. Player 1 wishes to maximise the result of the game while player 2 wishes to minimise it. Without *a priori* knowledge of the strategies employed by their opponents, the players calculate the worst possible outcome of any strategy they choose. Hence, player 1 locates the smallest payoff in each of the strategies  $i \in X$  and then finds the strategy corresponding to the highest payoff from amongst these values. Likewise, player 2 locates the largest payoff in each of the strategies  $j \in Y$  and then finds the strategy corresponding to the highest payoff from amongst these values.

Define  $\alpha : X \rightarrow \mathbb{R}$  and  $\beta : Y \rightarrow \mathbb{R}$  as

$$\alpha(x) = \min_{j \in Y} P(x, j) \quad (16)$$

$$\beta(y) = \max_{i \in X} P(i, y) \quad (17)$$

Define

$$\alpha^* = \max_{i \in X} \alpha(i) \quad (18)$$

$$\beta^* = \min_{j \in Y} \beta(j) \quad (19)$$

Trivially,  $\alpha^* \leq \beta^*$ . However, when  $\alpha^* = \beta^*$  the game has at least one pure strategy solution  $(x^*, y^*)$ , called an equilibrium point, saddle point or minimax point, such that  $P(i, y^*) \leq P(x^*, y^*) \leq P(x^*, j)$ ,  $\forall i \in X, j \in Y$  (Llewellyn et al., 1988). It is also true that if  $P(i, y^*) \leq P(x^*, y^*) \leq P(x^*, j)$ ,  $\forall i \in X, j \in Y$ , then  $\alpha^* = \beta^*$ . The payoff  $P(x^*, y^*)$  is called the value of the game.

### A.4 Mixed Strategies

In games without saddle point solutions we seek mixed strategy solutions, see von Neumann and Morgenstern's (1944) classic text for an introduction to the concepts of pure and mixed strategies for strategic normal-form games. That is, a linear combination of two or more strategies. Denote  $\mathcal{X}$  and  $\mathcal{Y}$  as the set of all normalised distributions over  $X$  and  $Y$  respectively. Define  $\mathcal{Q} : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}$  as the payoff function

$$\mathcal{Q}(\xi, \eta) = \sum_i \sum_j P(i, j) \xi_i \eta_j. \quad (20)$$



For games of the kind described in this Section, a mixed strategy solution is guaranteed to exist (Blackwell and Girshick, 1979, pp. 65–67). Note that  $\mathcal{X}$  and  $\mathcal{Y}$  contain the set of standard basis vectors  $\{\mathbf{e}_i | i \in X\}$  and  $\{\mathbf{e}_j | j \in Y\}$  respectively. Hence, the space of mixed strategy solutions includes elements which correspond to all of the pure strategy solutions.

For games of the type discussed in this section, there is a theorem (Blackwell and Girshick, 1979, p. 65) that states

$$P(\xi^*, \mathbf{e}_j) = P(\mathbf{e}_i, \eta^*) = P(\xi^*, \eta^*) = v^*, \quad (21)$$

for all  $i \in X$ ,  $j \in Y$ , where  $\xi^*$  and  $\eta^*$  are player 1's and player 2's respective mixed strategy solution and  $v^*$  is the value of the game. Furthermore, to calculate a solution to the game  $\xi^*$  and  $\eta^*$  we need only to follow the following steps. First, without loss of generality we scale the payoff function  $P$  by a sufficiently large constant  $k$  to ensure that all payoff values are strictly non-negative. Define  $\tilde{P}(\xi, \eta) = P(\xi, \eta) + k$  and denote the corresponding normal-form matrix by  $\tilde{M}$ . Next, we solve the two linear programs

$$\begin{array}{ll} \text{maximise} & v_x \\ \text{subject to} & \tilde{M}^\top \xi^* \geq v_x \mathbf{e}, \\ & \sum_i \xi_i^* = 1, \end{array} \quad (22)$$

and

$$\begin{array}{ll} \text{maximise} & v_y \\ \text{subject to} & \tilde{M} \eta^{*\top} \leq v_y \mathbf{e}, \\ & \sum_j \eta_j^* = 1, \end{array} \quad (23)$$

where  $\mathbf{e}$  denotes the column vector, of suitable size, with unit entries. At least one solution to the linear programs is guaranteed to exist such that  $v^* = v_x = v_y$ . The value of the game is then  $v^* - k$ .

The sets  $\mathcal{X}^*$  and  $\mathcal{Y}^*$  of all mixed strategy solutions for player 1 and player 2 respectively are non-empty, convex, closed and bounded. Furthermore, the two sets contain a finite number of extreme points. Hence, the sets are in fact the convex hulls of their extreme points (Blackwell and Girshick, 1979, pp. 67–69). Determining  $\mathcal{X}^*$  and  $\mathcal{Y}^*$  is then a simple matter of finding extreme points. It can be proven that, to locate these extreme points we need only to enumerate over all possible non-singular sub-matrices of  $\tilde{M}$  of size  $2 \times 2$  or larger and find the mixed strategy solutions of those sub games and then test  $\tilde{M}$  for saddle points. Note that because the sub-matrices are non-singular, the linear programs (22) and (23) have unique solutions.

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19. ABSTRACT This study presents a foundation for the comparative analysis of the various combined arms teaming in a simulated environment. The study consists of three stages. First, a discrete-event combat-simulation model of two opposing generic combined arms teams is developed. This model is used to study the relationships between six key attributes of combined arms teams: communication; detection; lethality; mobility; protection; and sustainment. Second, a genetic algorithm is embedded within the combat-simulator to evolve strategies for combined arms teams against a static opposing force. Finally, a two-population genetic algorithm is used to coevolve two opposing forces against each other. Games theory is used to analyse the results and to provide advice on the impact of adding, removing and replacing assets or capabilities within the teams. We conclude that diversity and specialisation within combined arms teams is essential to the Land force. Furthermore, no single combined arms team is sufficient to ensure a tactical victory on the battlefield against all potential opponents. A range of different options for constructing combined arms teams is required.					